Mann-Whitney U test

Testing the difference between two sets of data when the data are ranked (ordinal data) or for measured (interval data) when the data are not known to be normally distributed.

When is it used?

- Tests for the difference between 2 sets of data.
  - When the data are ranked or given scores (ordinal data).
  - When the data are accurately measured (interval data) but not known to be normally distributed.

Question

- A student investigated whether there were more trichomes (stings) on nettles that were grazed compared with nettles that were ungrazed.

- He counted the number of trichomes per cm^2 on a sample of nettle leaves from each area and applied a Mann-Whitney U test.

State the hypotheses

- Null hypothesis:
  - There **is no** significant difference between the number of trichomes from grazed & ungrazed leaves.

- Alternative hypothesis:
  - The number of trichomes on the grazed leaves are significantly higher than those on the ungrazed leaves.
    - Note that in this case we are predicting the direction of the difference rather than just saying they are different.

Tabulate the data

	Number of trichomes per cm^2 in each sample							
Grazed plants	12	14	15	17	19	22	23	26
Ungrazed plants	10	13	14	14	16	20	21	

It is easier if we write the data in order of increasing values

Rank order the data into one rank from smallest (rank 1) to largest

	Number of trichomes per cm^2 in each sample							
Grazed plants	12	14	15	17	19	22	23	26
Rank	2.5	6	8	11	12	13	14	15
Ungrazed plants	10	12	13	14	14	16	16	
Rank	1	2.5	4	6	6	9.5	9.5	

Take care with tied scores.

Add up the ranks for the smaller sample and call this R

	Number of trichomes per cm^2 in each sample								R
Grazed plants	12	14	15	17	19	22	23	26	
Rank	2.5	6	8	11	12	13	14	15	
Ungrazed plants	10	12	13	14	14	16	16		
Rank	1	2.5	4	6	6	9.5	9.5		38.5

If both samples are the same size nominate one to be R.

Calculate the value of U & U'

- n_1 = the size of the R sample (smallest) (in this case 7).
- n_2 = the size of the other sample (in this case 8).
- R = the sum of ranks of the R sample (in this case 38.5).

$$\mathbf{U = n_1 n_2 + \frac{n_1(n_1 + 1)}{2} - R}$$

$$\mathbf{U' = n_1 n_2 - U}$$

So…

$$U = n_1 n_2 + \frac{n_1(n_1 + 1)}{2} – R$$

$$U' = n_1 n_2 - U$$

U = 7x8 + 7x8/2 – 38.5

= 56 + 28 – 38.5

= 45.5

U' = 7x8 – U

= 56 – 45.5

= 10.5

Is U or U' the smallest?

- U = 45.5
- U' = 10.5

- U' is the smaller number.

Look in printed M-W-U tables

- Look for the critical value (U_{crit}) for the appropriate sample sizes.

- Reject the null hypothesis & accept the alternative hypothesis if the smaller of U & U’ is less than or equal to U_{crit}..

Significance: p=0.05 for two tailed, p=0.025 for one tailed

n_1 \ n_2	1	2	3	4	5	6	7	8	9	10
1	-	-	-	-	-	-	-	-	-	-
2	-	-	-	-	-	-	-	0	0	0
3	-	-	-	-	0	1	1	2	2	3
4	-	-	-	0	1	2	3	4	4	5
5	-	-	0	1	2	3	5	6	7	8
6	-	-	1	2	3	5	6	8	10	11
7	-	-	1	3	5	6	8	10	12	14
8	-	0	2	4	6	8	10	13	15	17
9	-	0	2	4	7	10	12	15	17	20

So…

- $U_{crit} = 10$
- $U' = 10.5$
- Cannot Reject H_0