Module 5 Newtonian World & Astrophysics

Module 5: Newtonian world and astrophysics

The aim of this module is to show the impact Newtonian mechanics has on physics. The microscopic motion of atoms can be modelled using Newton's laws and hence provide us with an understanding of macroscopic quantities such as pressure and temperature. Newton's law of gravitation can be used to predict the motion of planets and distant galaxies. In the final section we explore the intricacies of stars and the expansion of the Universe by analysing the electromagnetic radiation from space. As such, it lends itself to the consideration of how the development of the scientific model is improved based on the advances in the means of observation (HSW1, 2, 5, 6, 7, 8, 9, 11).

In this module, learners will learn about thermal physics, circular motion, oscillations, gravitational field, astrophysics and cosmology.

Module 5 Newtonian World & Astrophysics

Unit 3 Oscillations

5.3 Oscillations

Oscillatory motion is all around us, with examples including atoms vibrating in a solid, a bridge swaying in the wind, the motion of pistons of a car and the motion of tides. (HSW1, 2, 3, 5, 6, 8, 9, 10, 12)

This section provides knowledge and understanding of simple harmonic motion, forced oscillations and resonance.

You are here! →

Module 5 – Newtonian world and astrophysics

5.1 Thermal physics

5.2 Circular motion

5.3 Oscillations

5.4 Gravitational fields

5.5 Astrophysics and cosmology

Module 6 – Particles and medical physics

6.1 Capacitors

6.2 Electric fields

6.3 Electromagnetism

6.4 Nuclear and particle physics

6.5 Medical imaging

5.3 Oscillations

- 5.3.1 Simple Harmonic Oscillations
- 5.3.2 Energy of a Simple Harmonic Oscillator
- 5.3.3 Damping

5.3.1 Simple Harmonic Oscillations

5.3.1 Simple harmonic oscillations

Learning outcomes

Learners should be able to demonstrate and apply their knowledge and understanding of:

(a) displacement, amplitude, period, frequency, angular frequency and phase difference

(b) angular frequency ω; $\omega = \frac{2\pi}{T}$ or $\omega = 2\pi f$

(c) **(i)** simple harmonic motion; defining equation $a = -\omega^2 x$

(ii) techniques and procedures used to determine the period/frequency of simple harmonic oscillations

(d) solutions to the equation $a = -\omega^2 x$ e.g. $x = A\cos\omega t$ or $x = A\sin\omega t$

(e) velocity $v = \pm\omega\sqrt{A^2 - x^2}$ hence $v_{max} = \omega A$

(f) the period of a simple harmonic oscillator is independent of its amplitude (isochronous oscillator)

(g) graphical methods to relate the changes in displacement, velocity and acceleration during simple harmonic motion.

How can we describe an oscillating object?

Simple Harmonic Motion

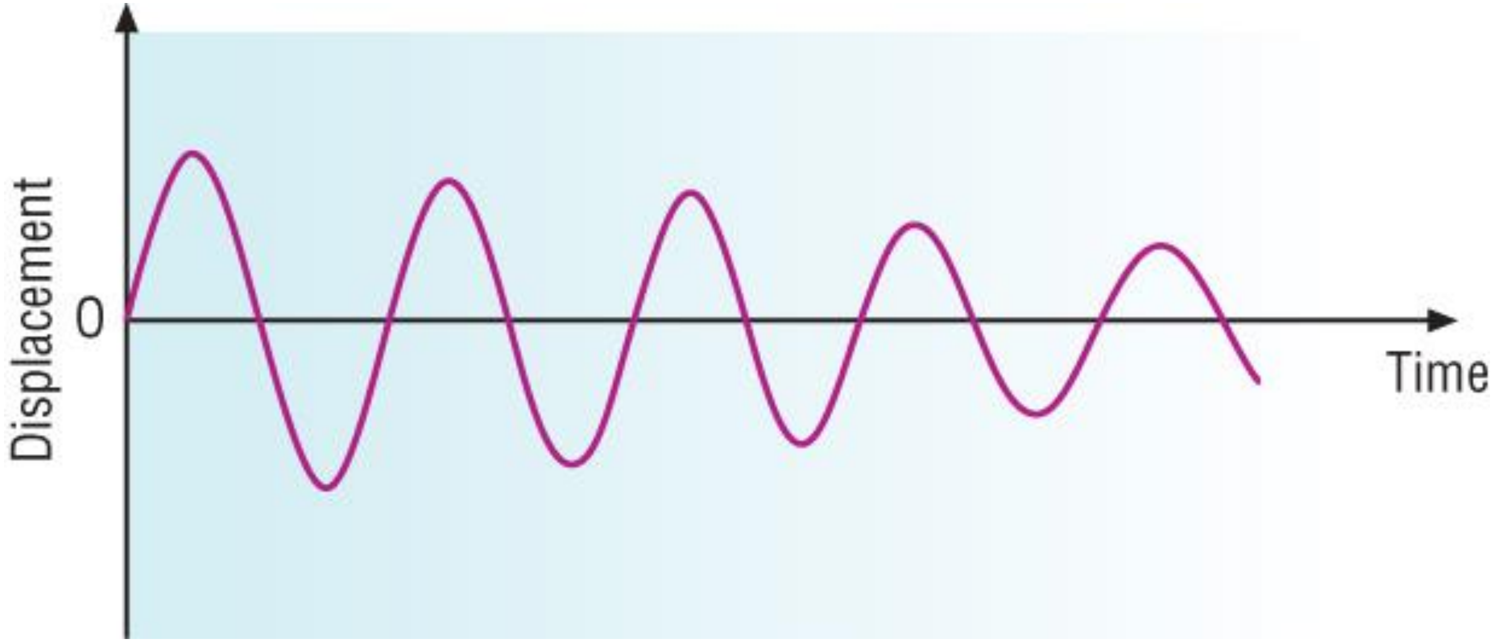
- The motion of a body which oscillates.
  - Simple is used to describe a lack of complicating factors (other forces).
    - Simple oscillations are smooth with constant amplitude, and the amplitude is not dependent on the period.
  - Harmonic is used because originally SHM was used to describe oscillating musical instruments in harmony.

Oscillations

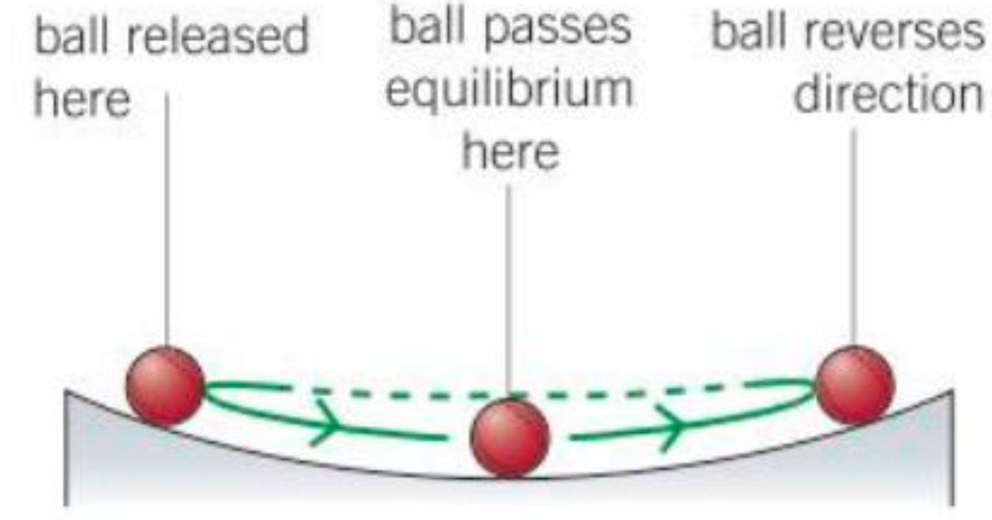
- More common than we think because:
  - – Many only happen for a very short time.
    - Eg. The sound made by knocking on a door
  - – Many are so fast that we can’t sense them.
    - Eg. Light waves
  - – Many take place in ways we can’t see.
    - Eg. Non visible Light spectrum (radio, micro, IR, UV, Xray, gamma)

Imagine a simple pendulum (a mass swinging on a length of string)

- What would a graph of displacement against time look like?



Describing oscillations



- Describe the motion of the ball using the terms:
  - Displacement
  - Amplitude
  - Period
  - Frequency

- Show these terms on a suitable graph

Describing Oscillations

- Many terms describing oscillations are also used to describe waves.
  - Obvious really – waves are often produced by oscillations.
    - Eg an oscillating string produces a sound wave.
- Displacement
  - (x) the distance in metres moved by the object from its mean (resting) position – can be positive or negative
- Amplitude
  - (A) the maximum displacement in metres – always positive
- Frequency
  - (f) the number of oscillations per second in Hertz (Hz)
- Period T
  - (T) the time in seconds taken for one complete oscillation

So,

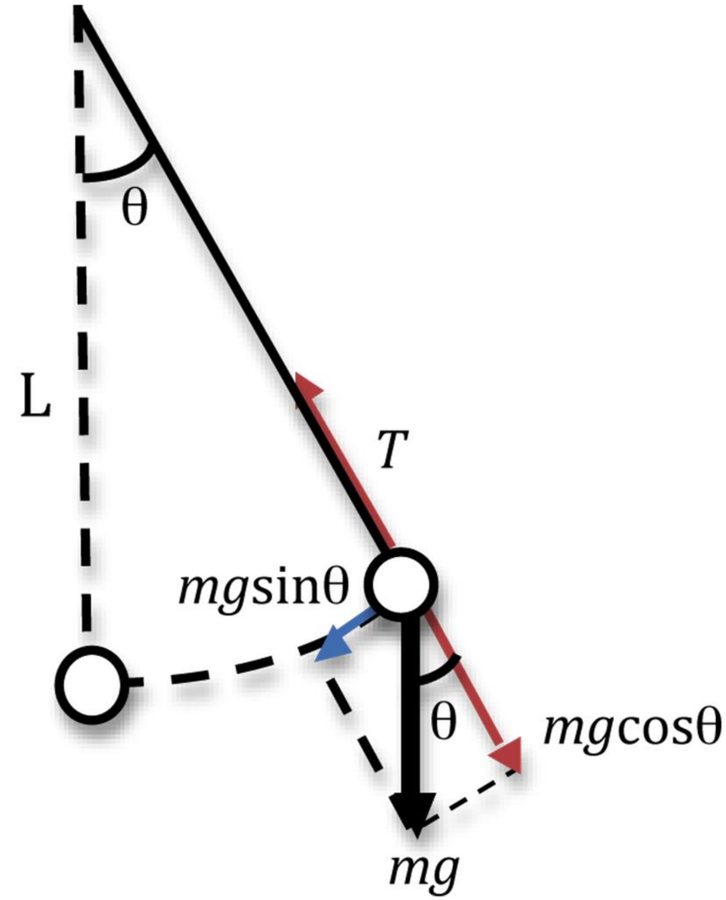
period = 1/frequency

Or

$T = 1/f$

Investigating a simple pendulum

- What are the variables that affect the time period for one complete swing?
  - Angle of displacement
  - Length of pendulum
  - Mass of the bob
- Select one, control the others and investigate its effect on oscillation time.
- Repeat for the other variables.



Results

- What did you find?
- The period of oscillation is:

$$T = 2\pi\sqrt{(l / g)}$$

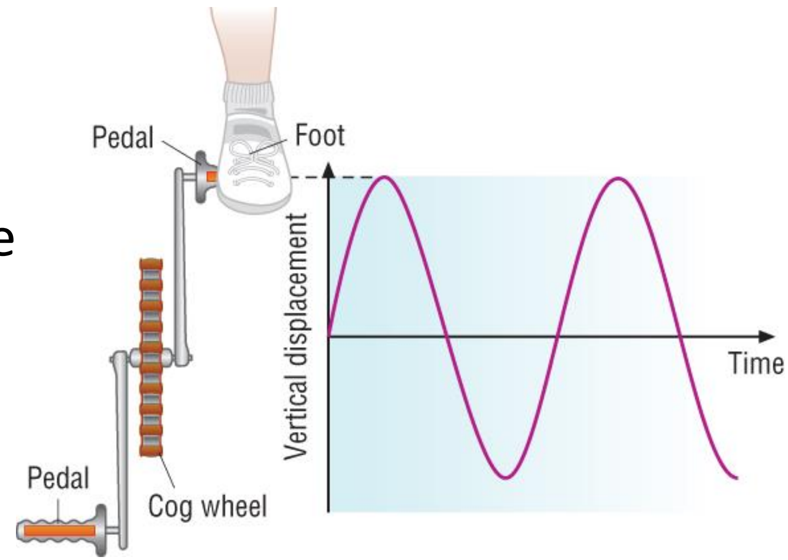
  - Where
    - T = Time period for a complete swing (s)
    - l = Length of pendulum (m)
    - g = acceleration of freefall (ms^{-2})

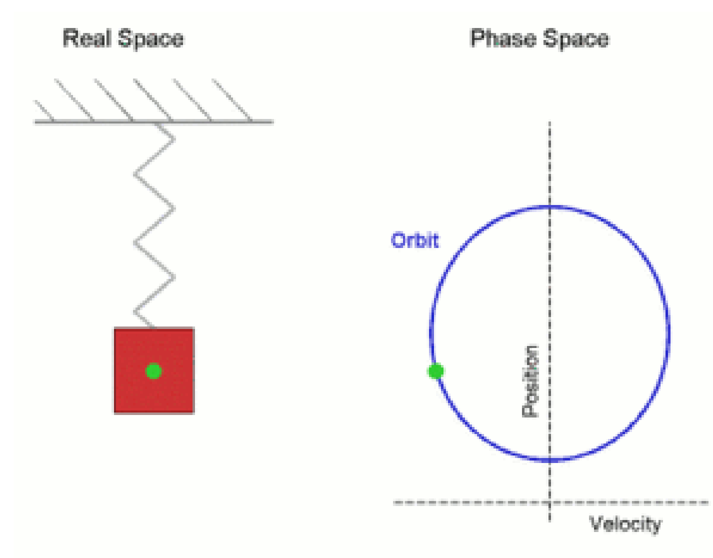
So the period of an oscillation is not affected by its amplitude.

We say it is **isochronous** (Greek for "same time")

Angular Frequency

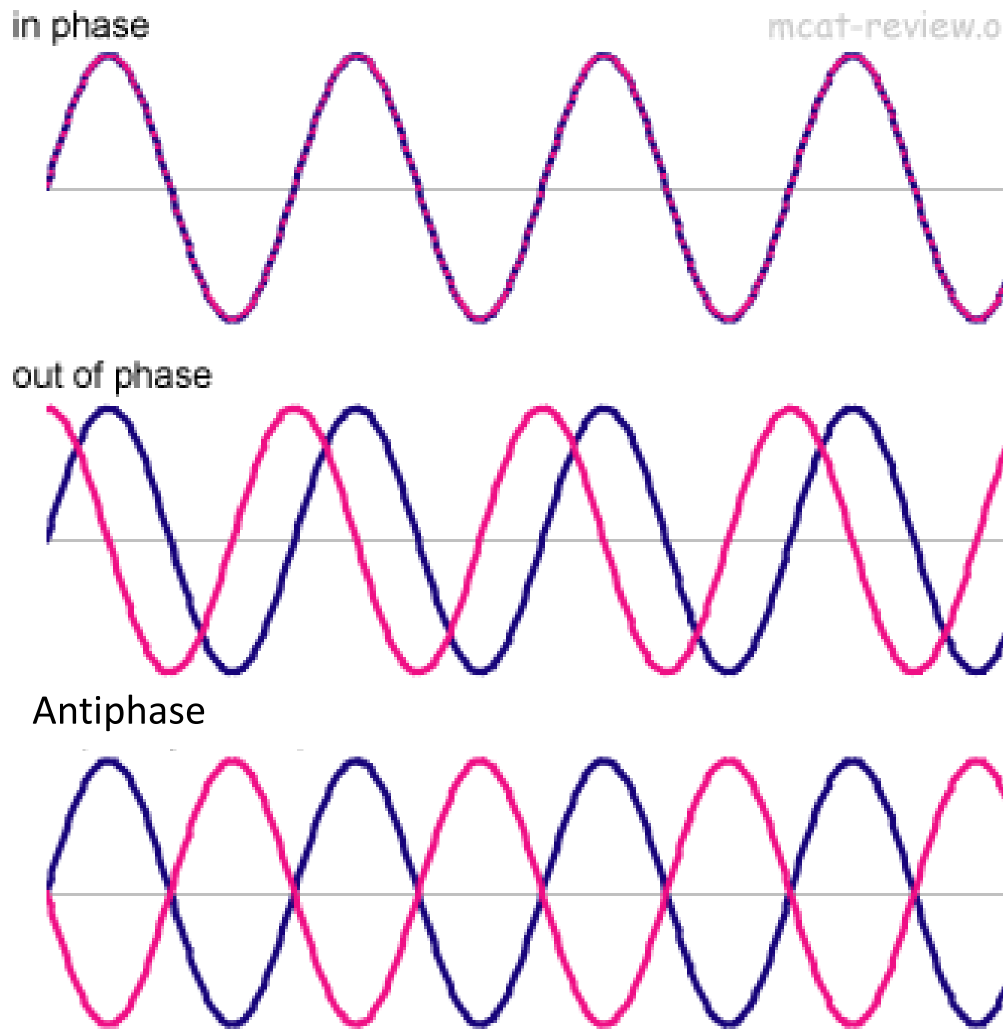
- Oscillations and circular motion are related.
  - We can compare one oscillating cycle with one rotation in a circle.
- Using angular measure, one oscillation corresponds to 2π radians.
- If there are f oscillations per second then the corresponding angle would be $2\pi f$ radians per second
  - This is the **angular frequency** ($2\pi f$) and has the symbol ω (omega)





Phase Difference

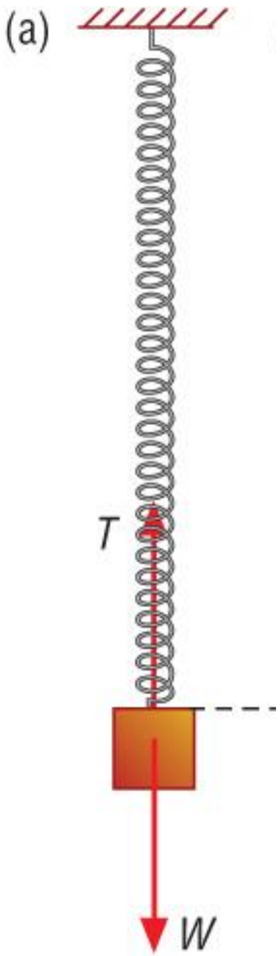
- The angle in radians between one oscillation and another.



Defining Simple Harmonic Motion

- Two statements define SHM.
  - An object undergoing Simple Harmonic Motion:
    - **Has an acceleration, a, which is proportional to its displacement, s, from a fixed point.**
    - **Accelerates in the opposite direction to its displacement.**

For example:



- At equilibrium:
  - The mass is supported by a spring with spring constant, k.
  - Tension = Weight.
- Stretched:
  - The upward force is T+kx.
  - Resultant force is kx upwards since T=W.
- Compressed:
  - Upward force is now T-ky.
  - Resultant force is ky downwards (-ky).
- In summary:
  - The resultant force is proportional to displacement but in opposite direction

In general equation form:

$$a = -cx$$

- Where
  - a = acceleration,
  - x = displacement,
  - c is a constant.

- **The constant, c, is the square of the angular frequency, ω.**

$$a = -\omega^2 x \quad \text{or} \quad a = -(2\pi f)^2 x$$

What else can we do with

$$a = -(2\pi f)^2 x$$

- This equation defines the acceleration of an oscillating object for a particular displacement.
  - If we differentiate this twice we can get equations relating the displacement of that object for a particular time.

- You do not need to know how to derive these equations but you do need to recognise them.

- The displacement equations are:

$$x = A\sin(2\pi ft) \qquad x = A\cos(2\pi ft)$$

Where A is the Amplitude of the oscillation

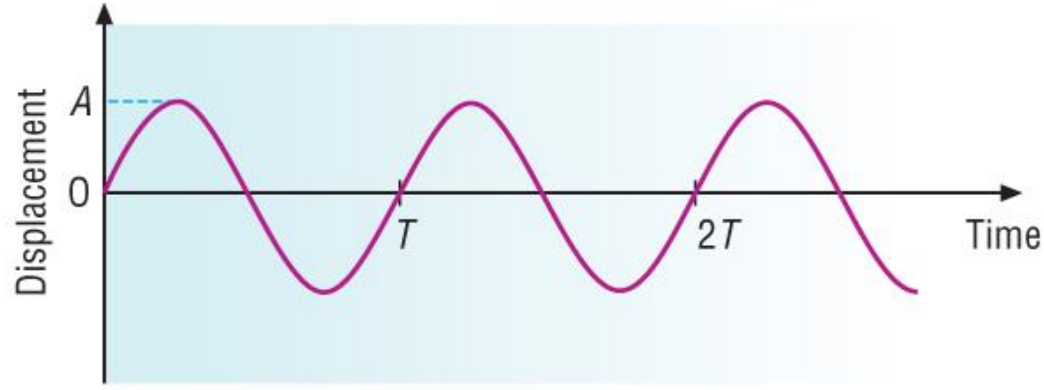
Recognise these equations

$$x = A\sin(2\pi f t) \qquad x = A\cos(2\pi f t)$$

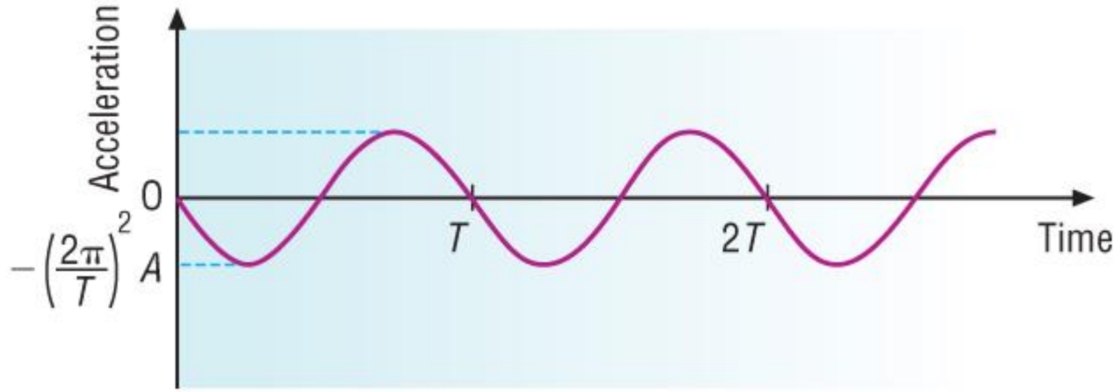
$$x = A\sin(\omega t) \qquad x = A\cos(\omega t)$$

Graphs of SHM

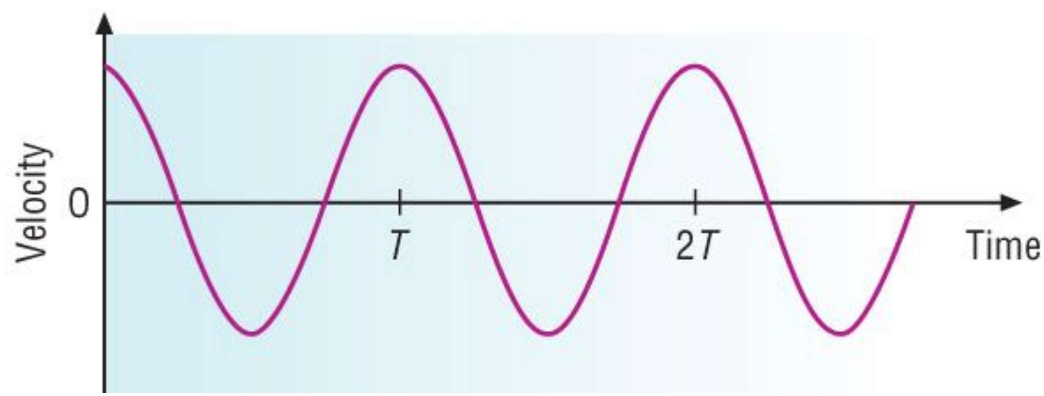
- x against t



- a against t
  - Since $a = -(2\pi/T)^2 x$

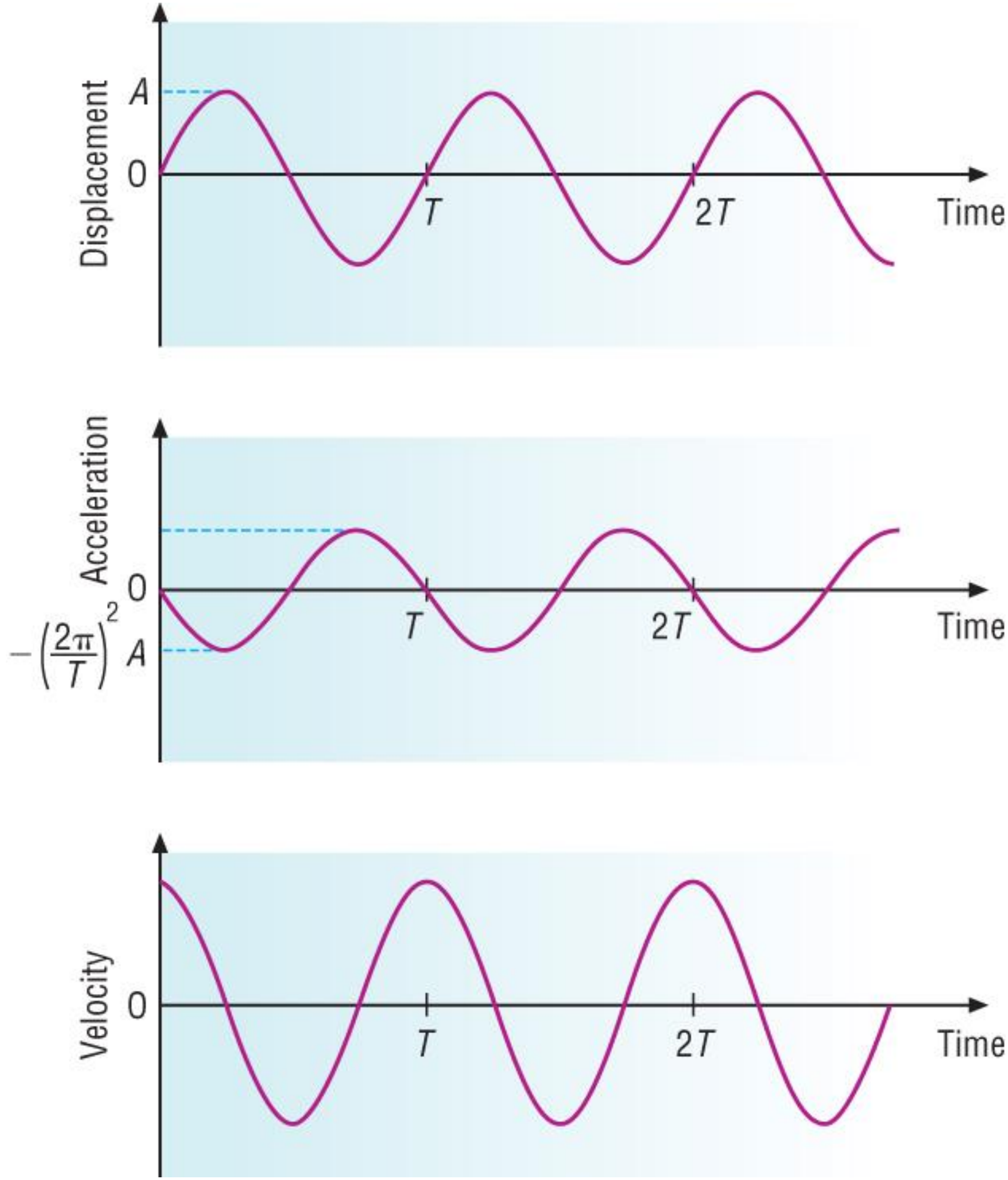


- v against t
  - Since v is the gradient of an x-t graph



Graphs of SHM

- Mark an X at the points on these graphs which show:
  a) Max velocity
  b) Max acceleration
  c) Zero velocity
  d) Zero acceleration



Velocity of an Oscillator

- The velocity of an oscillator will vary depending on:
  - – Its displacement.
    - Velocity will be zero at maximum displacement
    - Velocity will be maximum at zero displacement
  - – Its amplitude.
    - The higher the amplitude the further it has to travel in the same time so the faster it must go.

Velocity of an Oscillator

- The velocity of an oscillator can be calculated using:

$$v = \pm\omega\sqrt{A^2 - x^2}$$

The velocity for any particular displacement can be positive or negative depending on its direction.

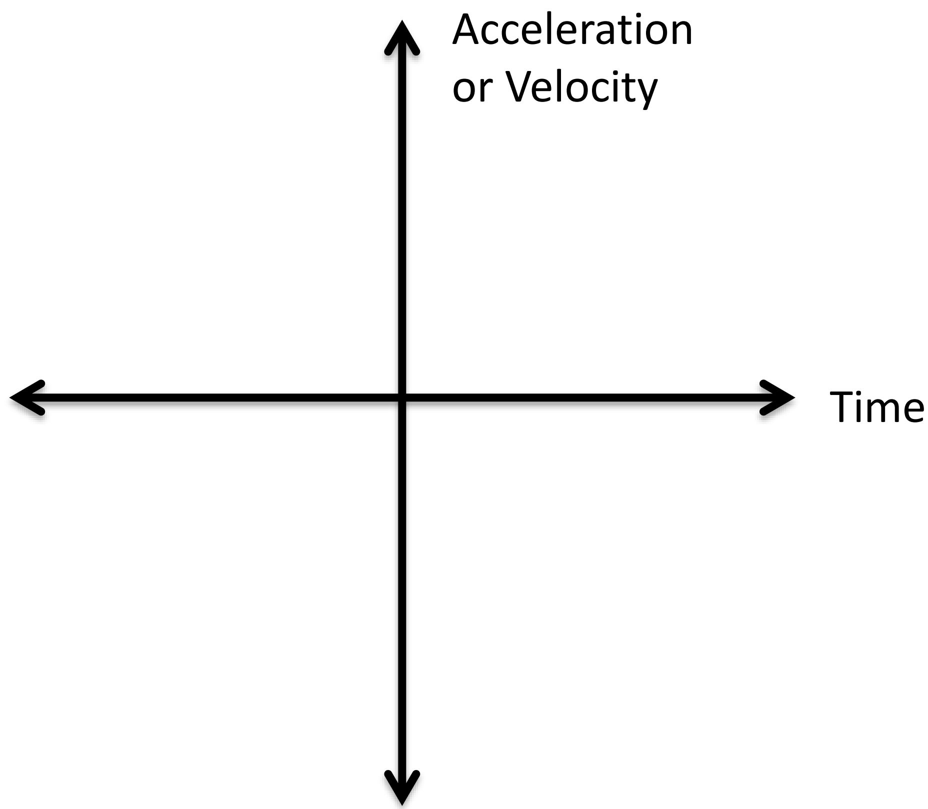
The velocity varies from zero (at x=A) to v_{max} (at x=0).

At v_{max} the equation becomes:

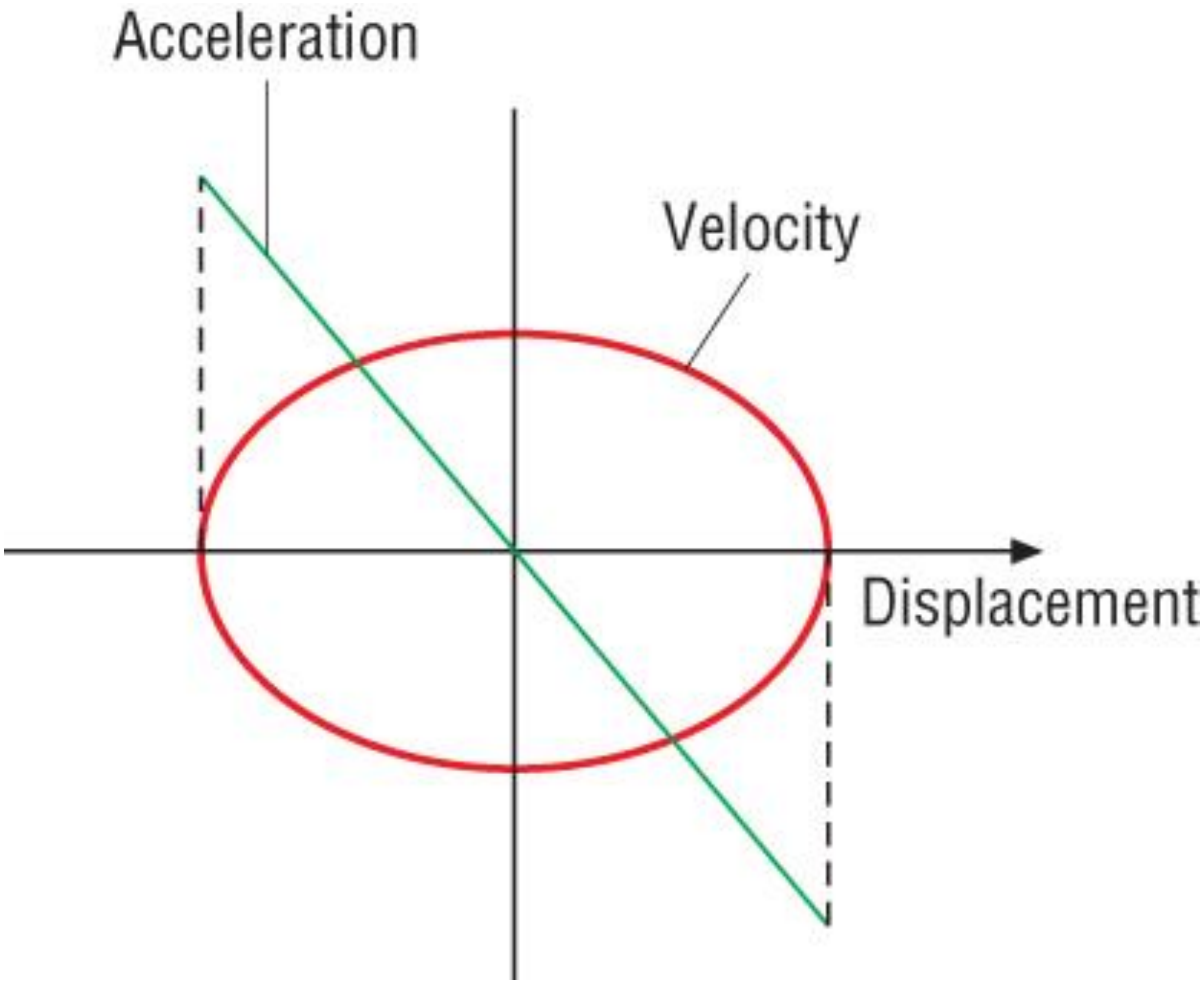
$$v_{max} = \pm\omega A$$

What would a graph of Acceleration against time look like?

Or velocity against time?



Describe why these graphs are like they are



- Points can't lie outside –A to +A
- There are two possible velocities for every displacement.
- Acceleration is always proportional and negative to displacement.

5.3.1 Simple Harmonic Oscillations (review)

5.3.1 Simple harmonic oscillations

Learning outcomes

Learners should be able to demonstrate and apply their knowledge and understanding of:

(a) displacement, amplitude, period, frequency, angular frequency and phase difference

(b) angular frequency ω; $\omega = \frac{2\pi}{T}$ or $\omega = 2\pi f$

(c) **(i)** simple harmonic motion; defining equation $a = -\omega^2 x$

(ii) techniques and procedures used to determine the period/frequency of simple harmonic oscillations

(d) solutions to the equation $a = -\omega^2 x$ e.g. $x = A\cos\omega t$ or $x = A\sin\omega t$

(e) velocity $v = \pm\omega\sqrt{A^2 - x^2}$ hence $v_{max} = \omega A$

(f) the period of a simple harmonic oscillator is independent of its amplitude (isochronous oscillator)

(g) graphical methods to relate the changes in displacement, velocity and acceleration during simple harmonic motion.

5.3.2 Energy of a Simple Harmonic Oscillator

5.3.2 Energy of a simple harmonic oscillator

Learning outcomes

Learners should be able to demonstrate and apply their knowledge and understanding of:

(a) interchange between kinetic and potential energy during simple harmonic motion

(b) energy-displacement graphs for a simple harmonic oscillator

How can we represent the energy transfers in a SHM?

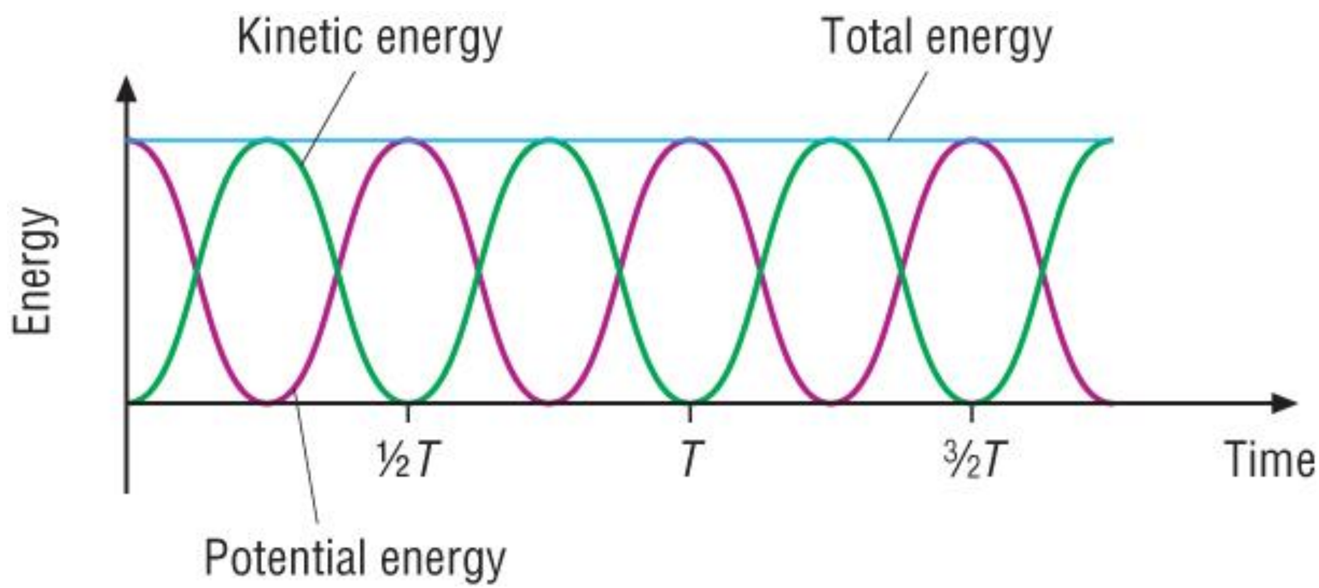
Energy in SHM

- With free oscillations there is no loss of energy to the surroundings.
  - We know this isn't true for most oscillations because over time there is a reduction in amplitude.

- However, there are energy changes taking place in all oscillations.

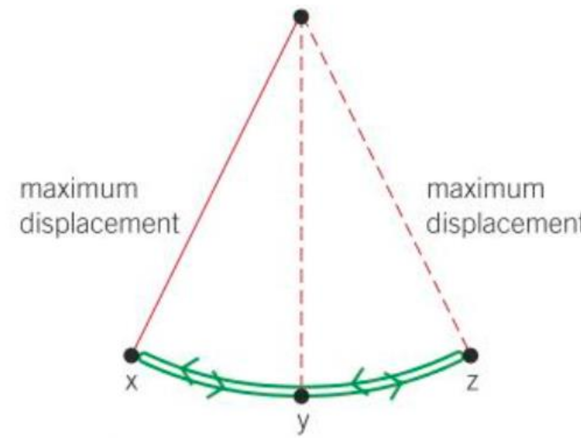
Energy changes in a pendulum

- A bob gains and loses kinetic energy as its velocity increases and decreases respectively.
- At the same time it will lose and gain gravitational potential energy as the bob loses and gains height.
- The period for both of these energy changes must be equal.

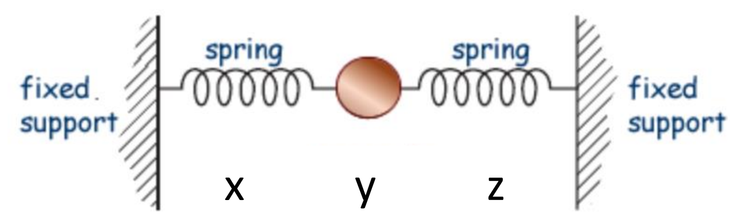


Transfer of Potential & Kinetic Energies

- Total energy remains constant.
- Potential energy can be in different forms:
  - Gravitational for swinging pendula.
  - Elastic potential for masses on a spring.

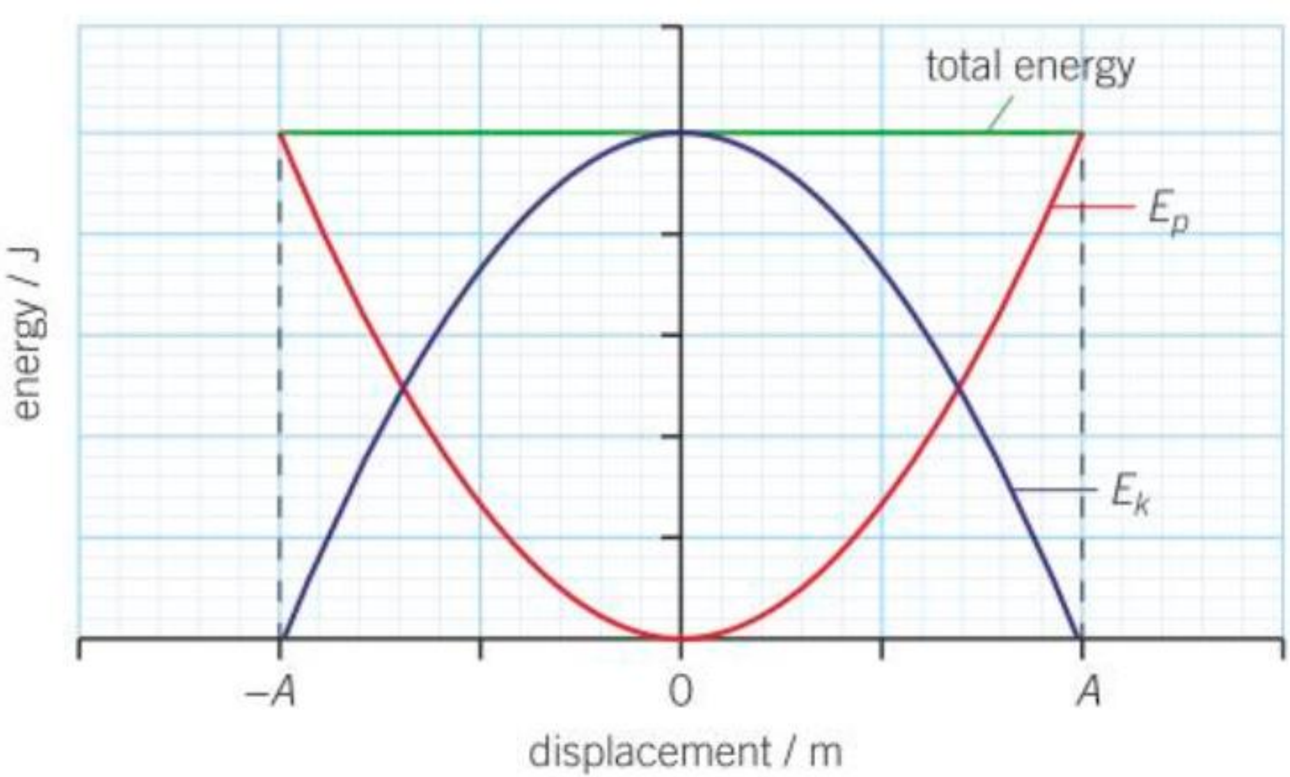


Position	E_p	E_k
x	E_{TOTAL}	0
y	0	E_{TOTAL}
z	E_{TOTAL}	0



Sketch a graph of energy against displacement

- Include kinetic, potential & total energies.



Calculating E_k in spring oscillators

- Elastic potential energy equation is:

$$E_p = \frac{1}{2}kx^2$$

- E_p is zero when x=0.

- E_p is maximum when x=A.

$$E_p = \frac{1}{2}kA^2$$

- E_k at any time is $E_{p(max)}$ - E_p

$$E_k = \frac{1}{2}kA^2 - \frac{1}{2}kx^2 = \frac{1}{2}k(A^2 - x^2)$$

5.3.2 Energy of a Simple Harmonic Oscillator (review)

5.3.2 Energy of a simple harmonic oscillator

Learning outcomes

Learners should be able to demonstrate and apply their knowledge and understanding of:

(a) interchange between kinetic and potential energy during simple harmonic motion

(b) energy-displacement graphs for a simple harmonic oscillator

5.3.3 Damping

5.3.3 Damping

Learning outcomes

Learners should be able to demonstrate and apply their knowledge and understanding of:

(a) free and forced oscillations

(b) **(i)** the effects of damping on an oscillatory system

(ii) observe forced and damped oscillations for a range of systems

(c) resonance; natural frequency

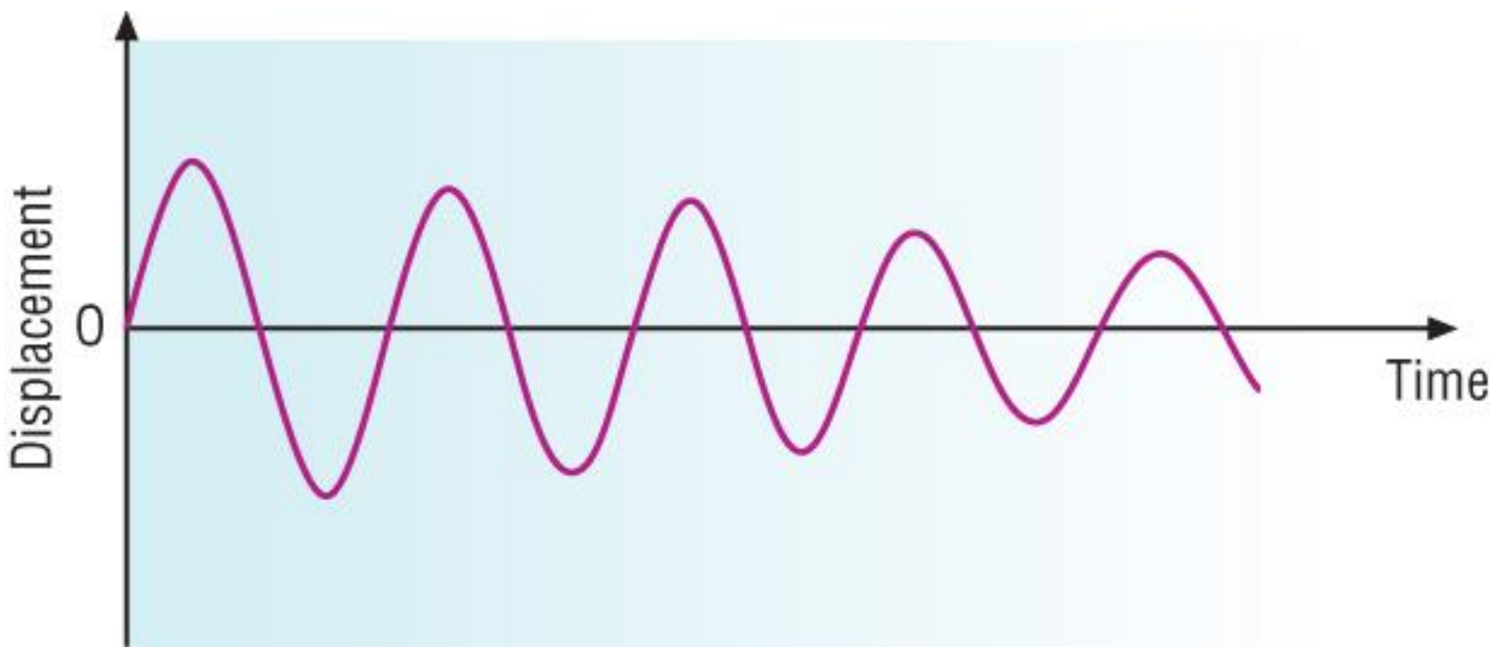
(d) amplitude-driving frequency graphs for forced oscillators

(e) practical examples of forced oscillations and resonance.

Damping? I thought that was about getting wet.

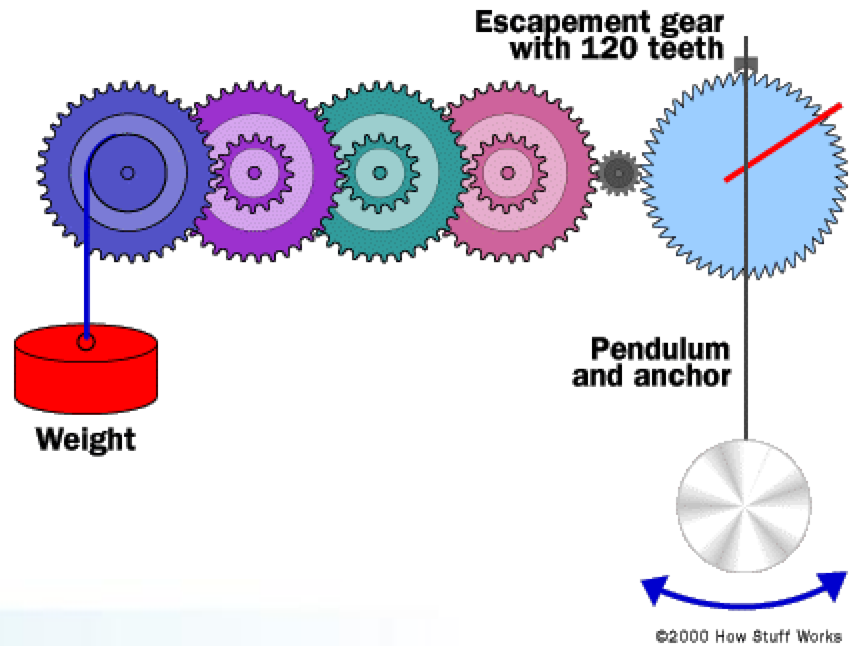
Imagine a simple pendulum (a mass swinging on a length of string)

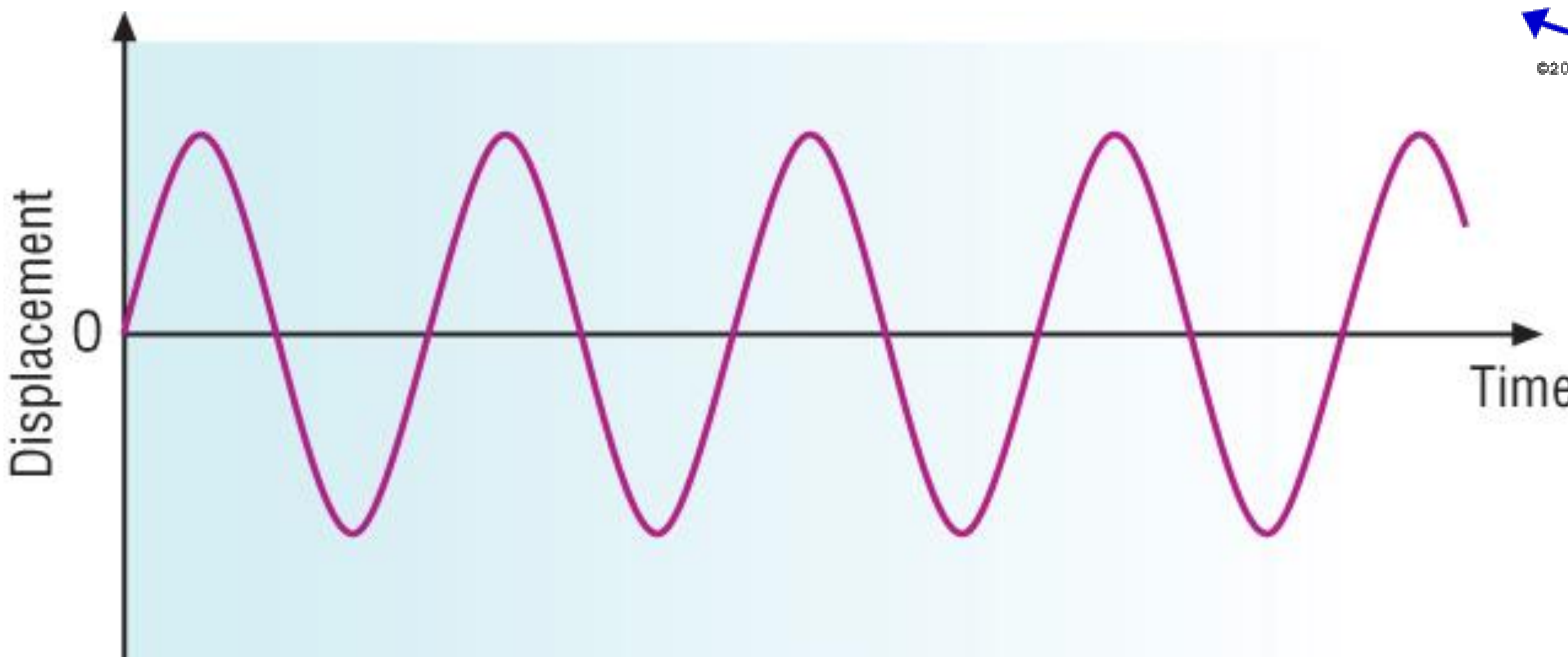
- What would a graph of displacement against time look like?



The pendulum in a clock is attached to a drive mechanism

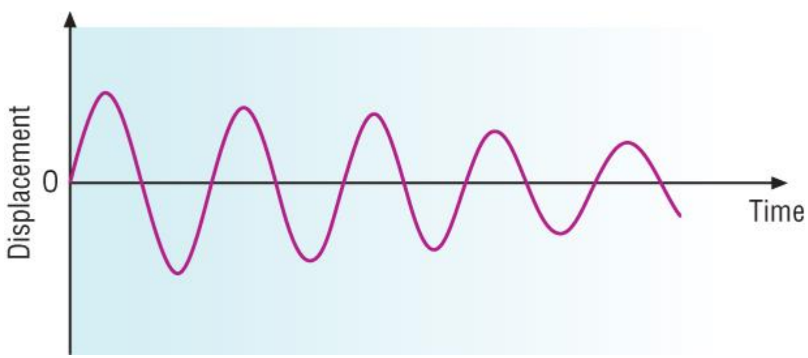
- What would it's displacement-time graph look like?

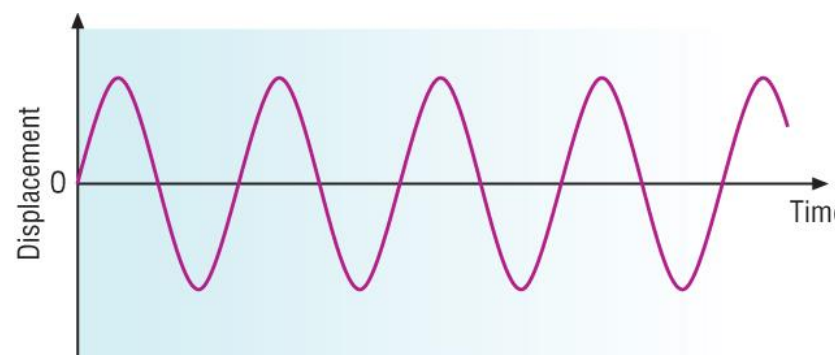




Free Oscillations

- Free oscillations do not have driving mechanisms but are not affected by friction either.
- Which trace would they have?



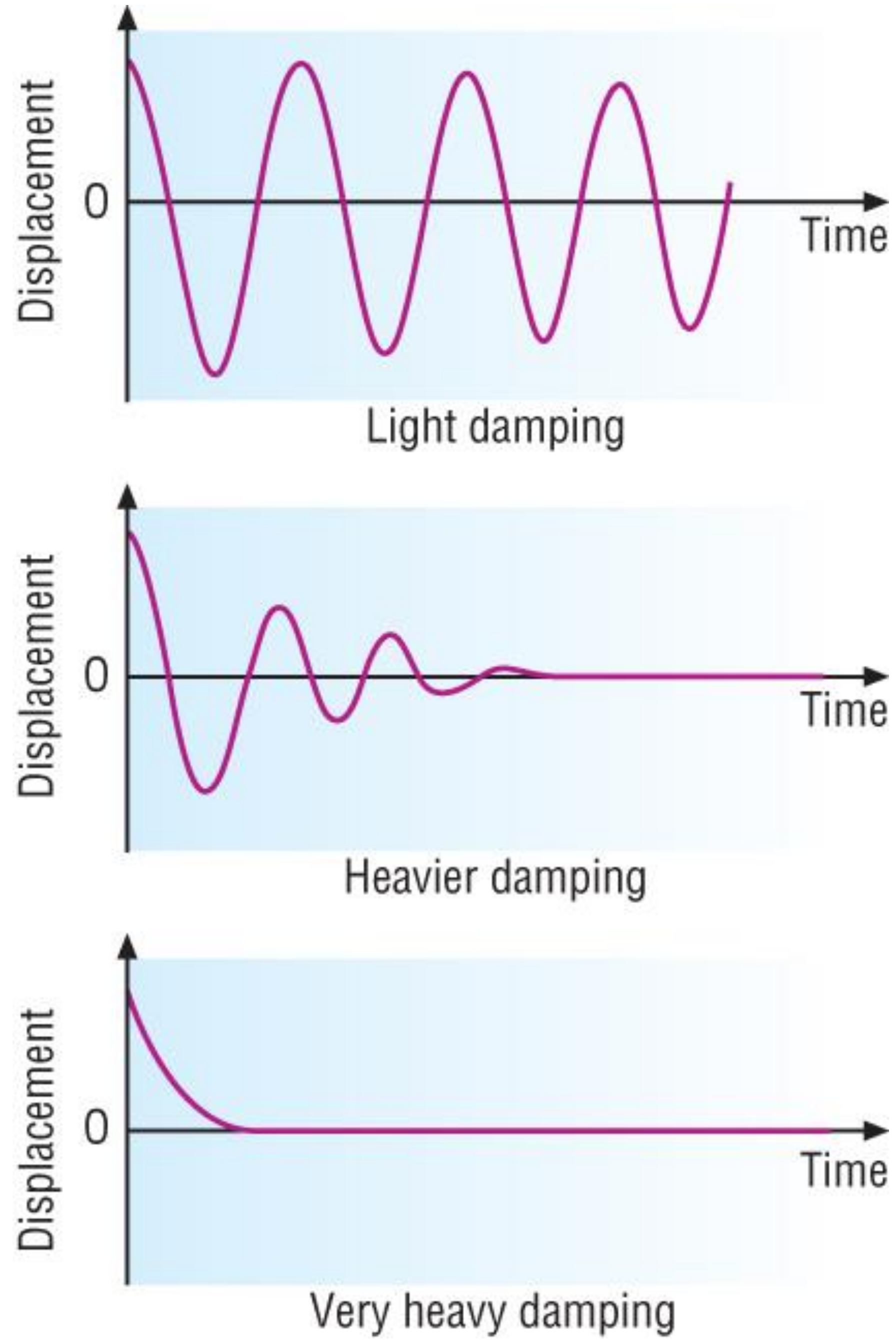


Free or Forced

- Free oscillations:
  - Free of any external forces (inc. friction)
  - The frequency of the oscillation is known as the natural frequency.
- Forced oscillations:
  - A periodic driver force is applied.
  - The frequency of the oscillation will be the same as the frequency of the driver force and is known as the driving frequency.
- Resonance:
  - If the driving frequency is the same as an object’s natural frequency the object will resonate.
    - The amplitude of oscillations increase dramatically

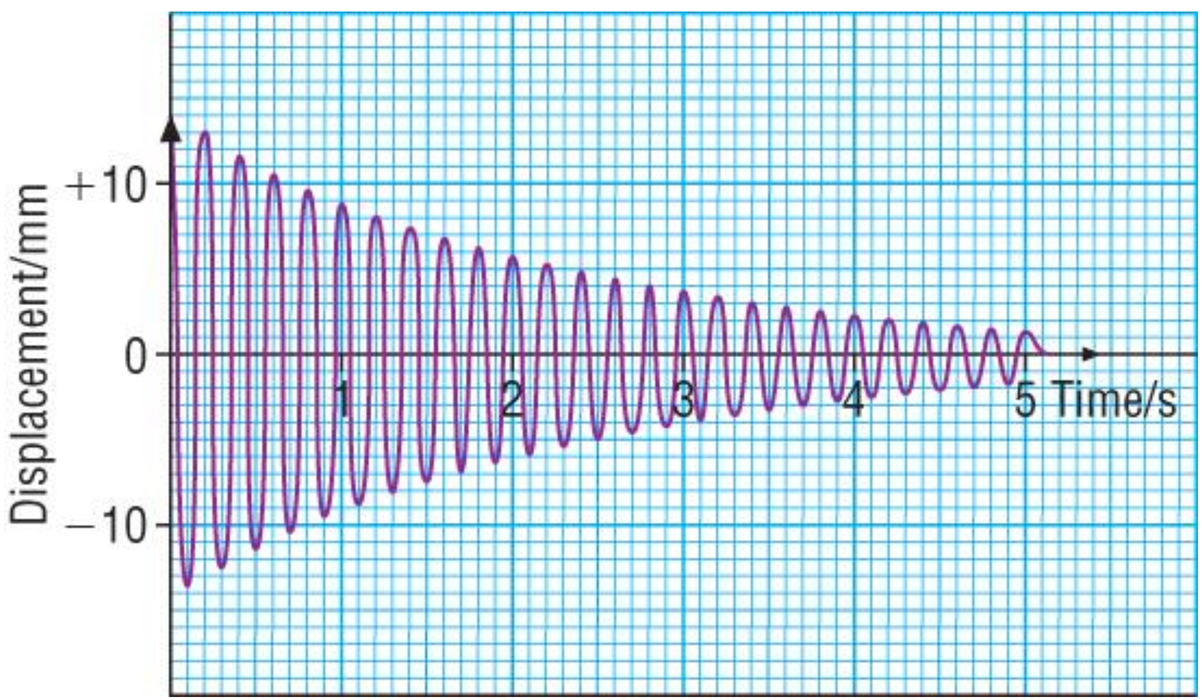
Damping

- In all oscillations except those which are totally free or driven, the amplitude will reduce over time.
- Damping is the process of deliberately reducing the amplitude.



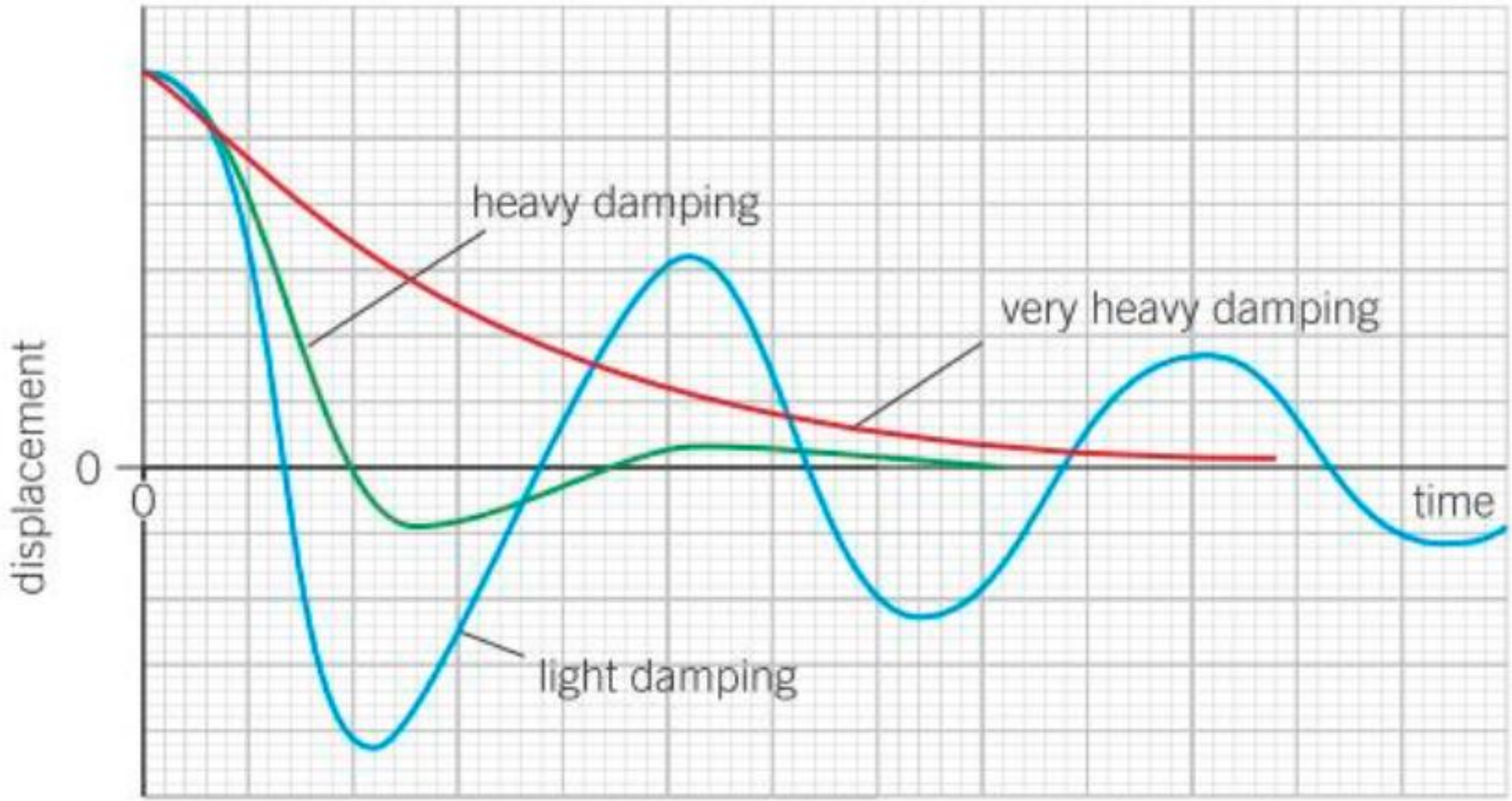
Exponential Decay of Amplitude

- Exponential decay results in the amplitude of oscillation decreasing by a fixed proportion in a given time.
- Here, the amplitude decreases by half from 14mm to 7mm in 1.6s. After another 1.6s the amplitude has halved again to 3.5mm.
- Light damping often gives this pattern.

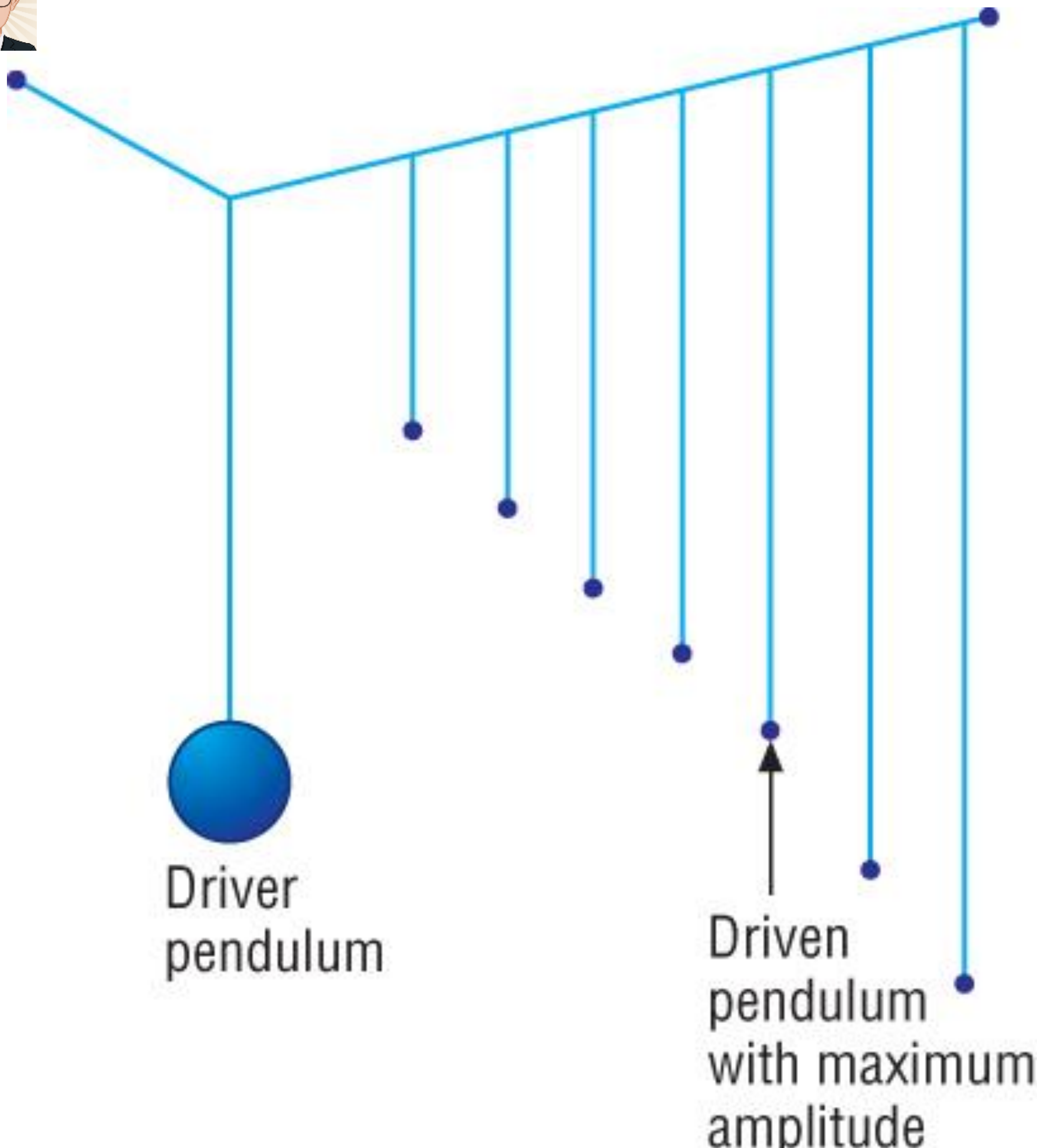


What can dampen an oscillation?

- Drag forces opposing the direction of movement.
  - Air resistance
  - Water resistance
  - Friction



▲ Figure 2 *The effects of different types of damping*



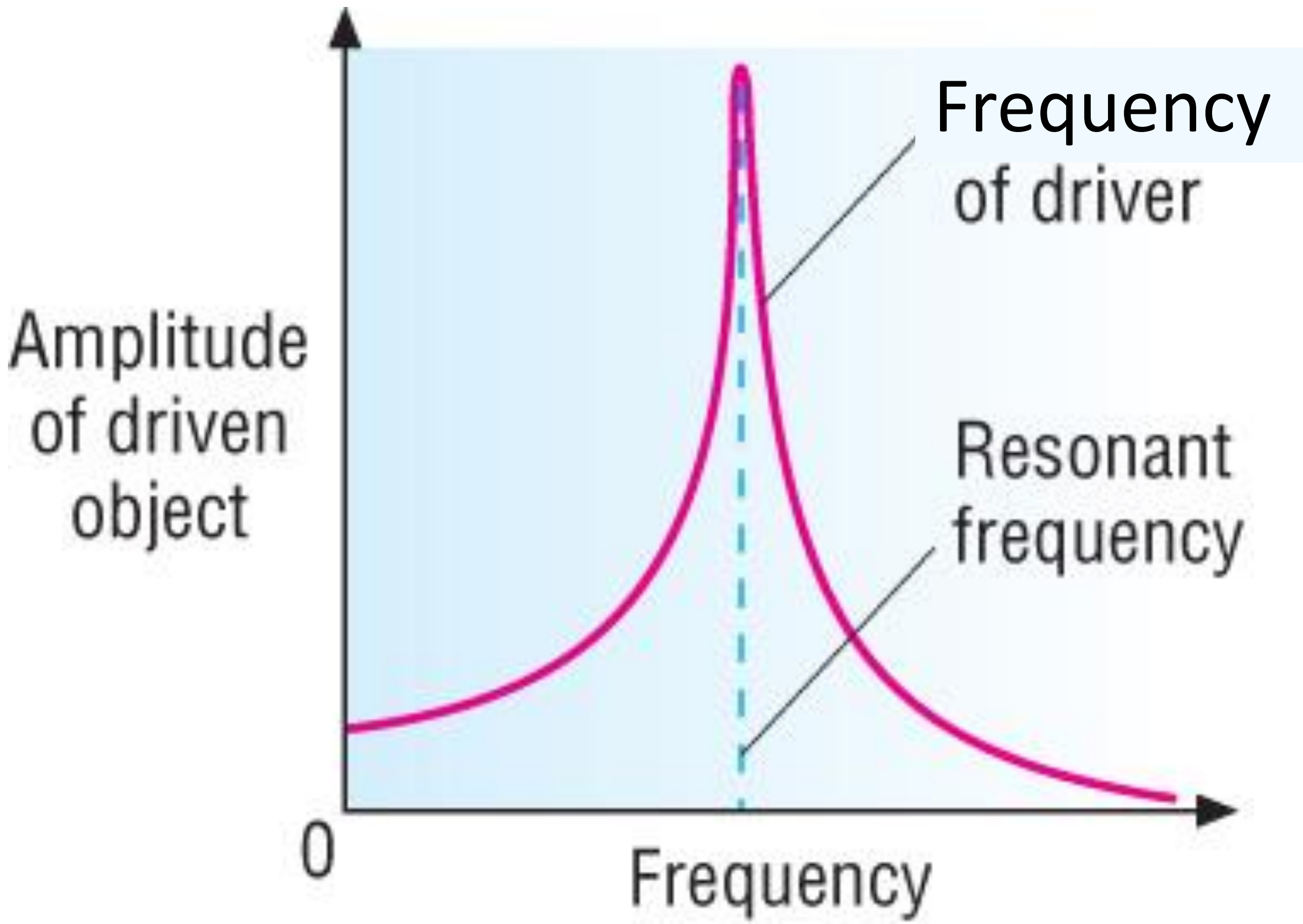
Barton’s Pendula

The driven pendulum with the highest amplitude is the one whose natural frequency matches the driver frequency.

Graphs of Resonance

A variable frequency driver can be used to cause the driven object to oscillate.

The amplitude of the driven object can be many times greater than that of the driver.

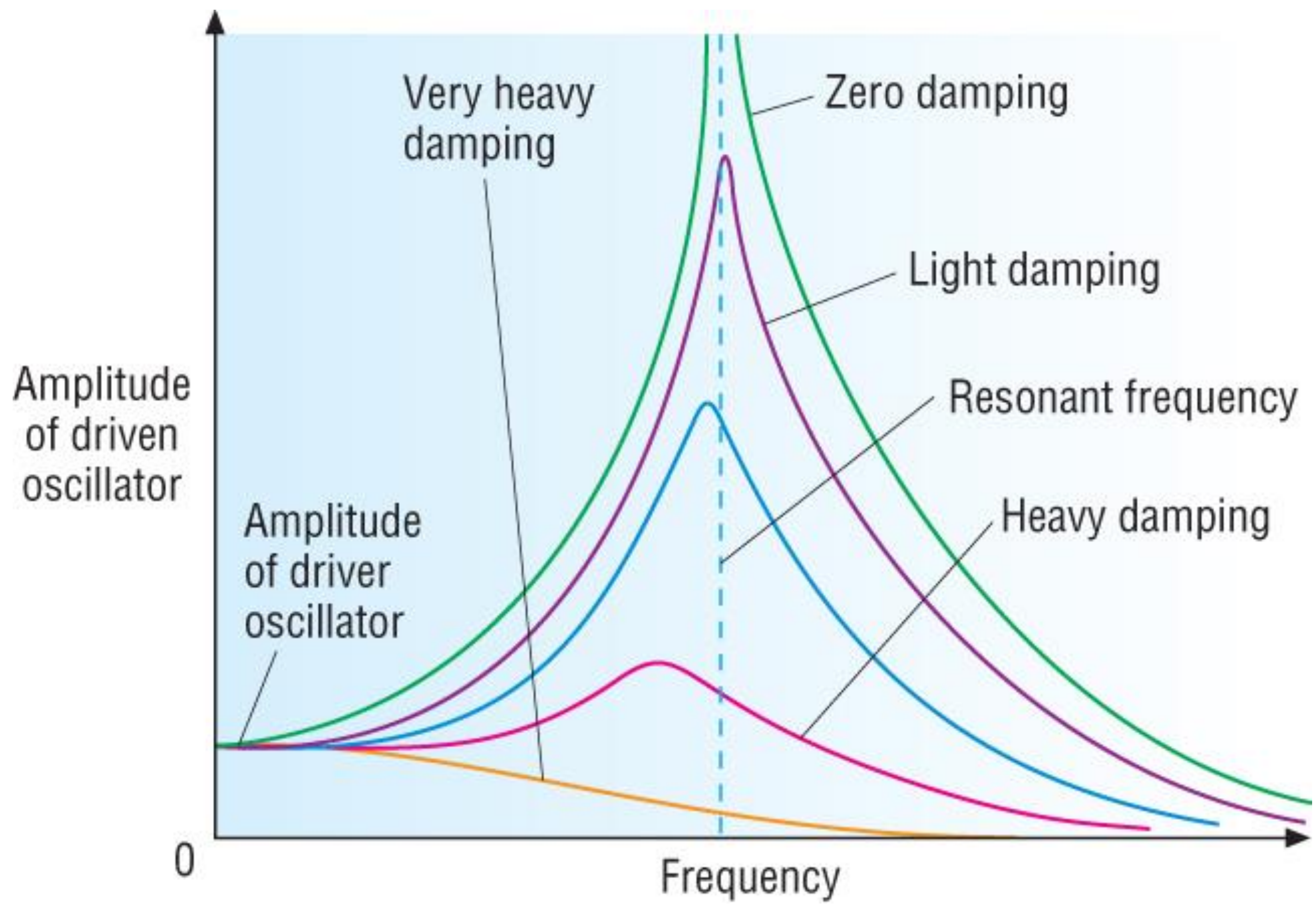


How does damping affect resonant oscillation?

Damping has two effects:

Reduces the amplitude of the driven oscillation

Slightly reduces the frequency of the driven oscillation



Uses of Resonance

- Resonating electrical circuits.
  - Tuning a radio/TV is to adjust the resonant frequency of the receiver to match the frequency of the transmitted signal.
- Resonating atomic nuclei.
  - Oscillating magnetic fields can resonate atomic nuclei, causing them to emit radio waves which can be detected in an MRI scanner.
- Resonating water molecules.
  - Microwave ovens emit EM radiation which causes water molecules to vibrate and so release heat.

Drawbacks of Resonance

- A rattle/vibration in a car which is running at a particular speed.
- The “feedback” squeak from PA systems.
- The wobbling Millennium Bridge.
  - Many footsteps drove the resonance of this bridge and caused it to wobble.
- Poor quality sound from cheap speakers.

5.3.3 Damping (review)

5.3.3 Damping

Learning outcomes

Learners should be able to demonstrate and apply their knowledge and understanding of:

(a) free and forced oscillations

(b) **(i)** the effects of damping on an oscillatory system

(ii) observe forced and damped oscillations for a range of systems

(c) resonance; natural frequency

(d) amplitude-driving frequency graphs for forced oscillators

(e) practical examples of forced oscillations and resonance.

Module 5 – Newtonian world and astrophysics

Complete! →

5.1 Thermal physics

5.2 Circular motion

5.3 Oscillations

5.4 Gravitational fields

5.5 Astrophysics and cosmology

Module 6 – Particles and medical physics

6.1 Capacitors

6.2 Electric fields

6.3 Electromagnetism

6.4 Nuclear and particle physics

6.5 Medical imaging