



Module 5 Newtonian World & Astrophysics

Module 5: Newtonian world and astrophysics

The aim of this module is to show the impact Newtonian mechanics has on physics. The microscopic motion of atoms can be modelled using Newton's laws and hence provide us with an understanding of macroscopic quantities such as pressure and temperature. Newton's law of gravitation can be used to predict the motion of planets and distant galaxies. In the final section we explore the intricacies of stars and the expansion of the Universe by analysing the

electromagnetic radiation from space. As such, it lends itself to the consideration of how the development of the scientific model is improved based on the advances in the means of observation (HSW1, 2, 5, 6, 7, 8, 9, 11).

In this module, learners will learn about thermal physics, circular motion, oscillations, gravitational field, astrophysics and cosmology.



Module 5 Newtonian World & Astrophysics

Unit 1 Thermal Physics

5.1 Thermal physics

This section provides knowledge and understanding of temperature, matter, specific heat capacity and specific latent heat with contexts involving heat transfer and change of phase (HSW1, 2, 5, 7).

Experimental work can be carried out to safely investigate specific heat capacity of materials (HSW4).

It also provides an opportunity to discuss how Newton's laws can be used to model the behaviour of gases (HSW1) and significant opportunities for the analysis and interpretation of data (HSW5).



Module 5 – Newtonian world and astrophysics

You are here! →

- 5.1 Thermal physics
- 5.2 Circular motion
- 5.3 Oscillations
- 5.4 Gravitational fields
- 5.5 Astrophysics and cosmology

Module 6 – Particles and medical physics

- 6.1 Capacitors
- 6.2 Electric fields
- 6.3 Electromagnetism
- 6.4 Nuclear and particle physics
- 6.5 Medical imaging



5.1 Thermal Physics

- 5.1.1 Temperature
- 5.1.2 Solid, Liquid & Gas
- 5.1.3 Thermal Properties of Materials
- 5.1.4 Ideal Gases



5.1.1 Temperature

5.1.1 Temperature

Learning outcomes

Learners should be able to demonstrate and apply their knowledge and understanding of:

- (a) thermal equilibrium
- (b) absolute scale of temperature (i.e. the thermodynamic scale) that does not depend on property of any particular substance
- (c) temperature measurements both in degrees Celsius ($^{\circ}\text{C}$) and in kelvin (K)
- (d) $T(\text{K}) \approx \theta(^{\circ}\text{C}) + 273.$



Is temperature
the same thing
as heat?



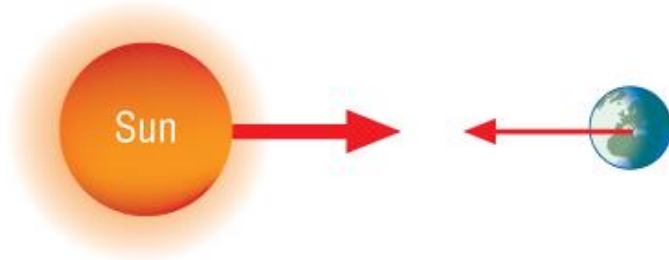
Thermal Equilibrium

- Two objects at the same temperature are said to be in **thermal equilibrium**.
- To reach thermal equilibrium, a resultant thermal energy transfer is made between the hotter object to the colder.

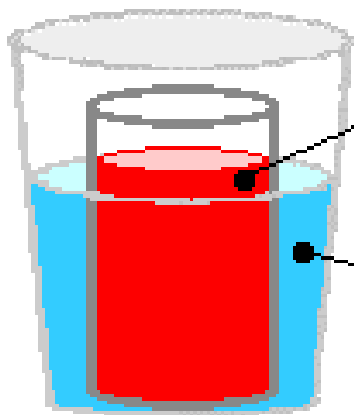


Resultant Thermal Energy Transfers

- Resultant (or net) energy is transferred between hotter objects to colder ones.



The Sun is hotter (at a higher temperature) than Earth so more energy is transferred from Sun to Earth than the other way round.

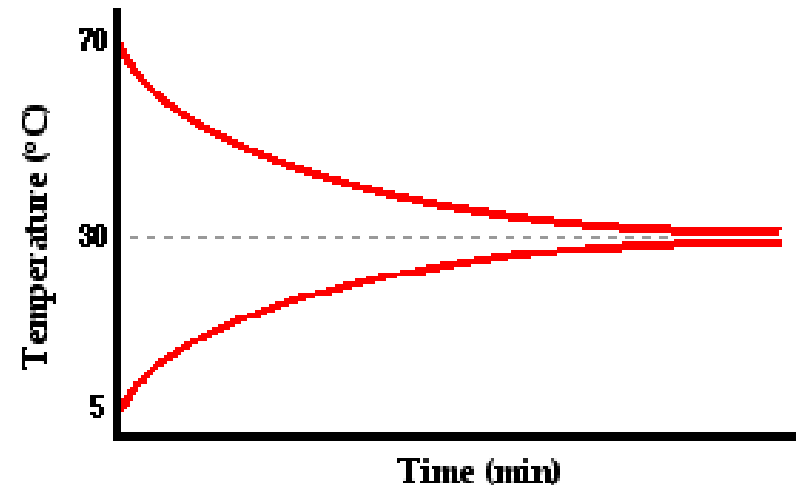


● Hot temperature
water inside a
metal can

High
 KE_{rand}

● Cold temperature
water inside a
Styrofoam cup

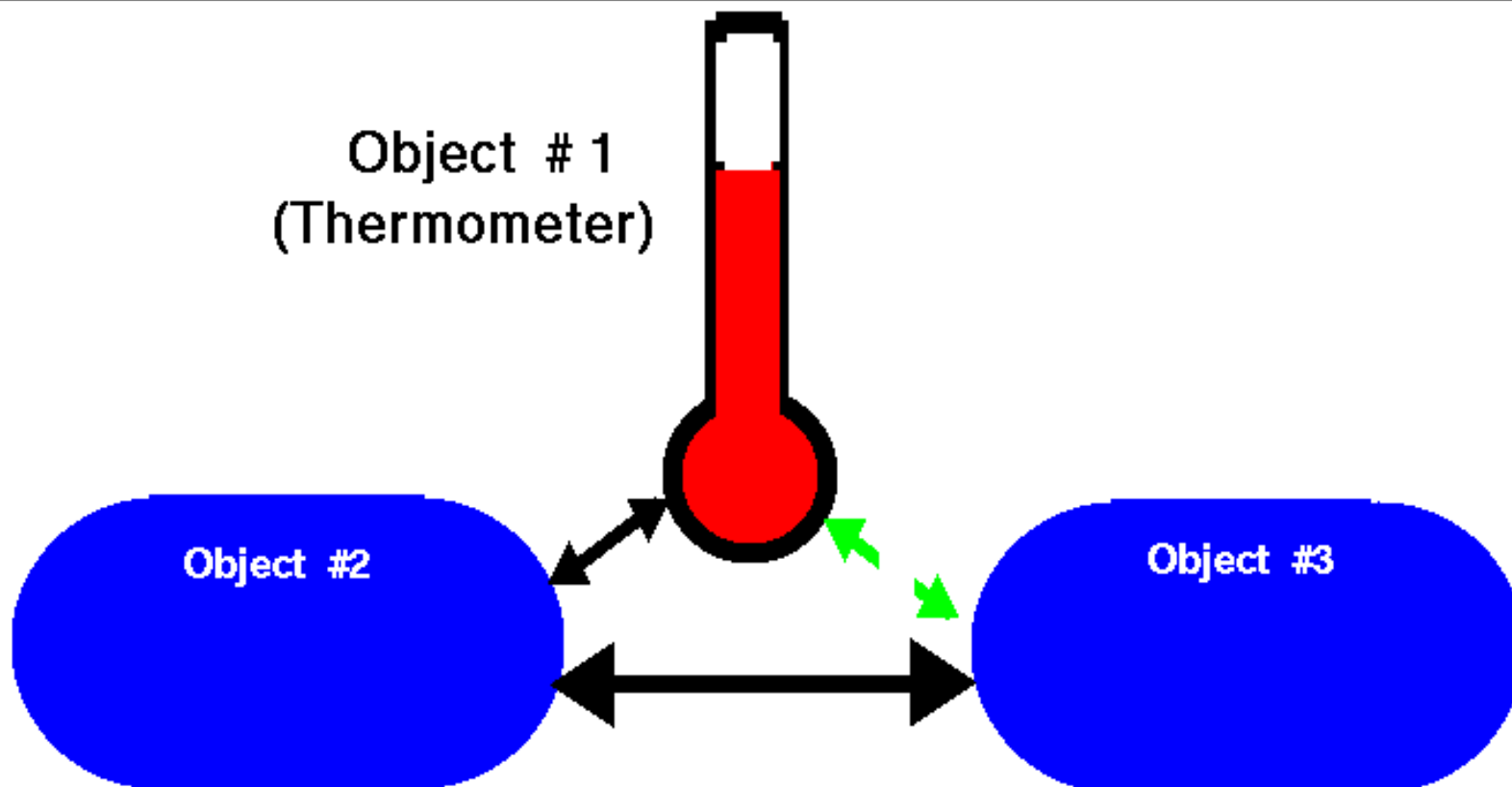
Low
 KE_{rand}





Thermodynamic Equilibrium ***(Zeroth Law)***

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When two objects are separately in thermodynamic equilibrium with a third object, they are in equilibrium with each other.

Objects in thermodynamic equilibrium have the same temperature.



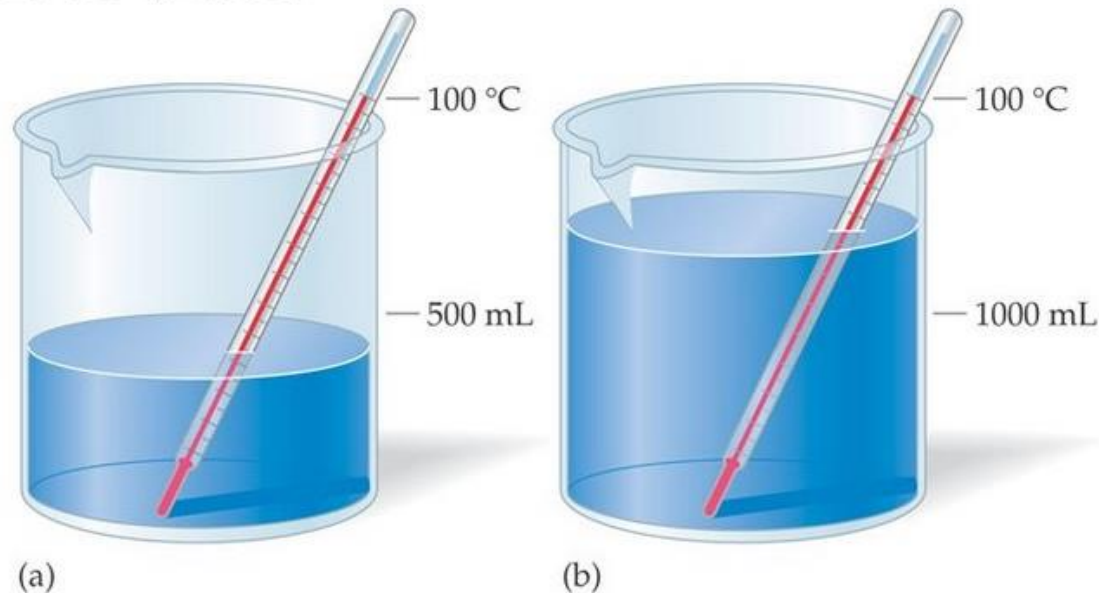
Laws of Thermodynamics

- You do not need to know these but...
 - **Zeroth law of thermodynamics** – If two thermodynamic systems are each in thermal equilibrium with a third, then they are in thermal equilibrium with each other.
 - **First law of thermodynamics** – Energy can neither be created nor destroyed. It can only change forms. In any process, the total energy of the universe remains the same. For a thermodynamic cycle the net heat supplied to the system equals the net work done by the system.
 - **Second law of thermodynamics** – The entropy (disorder) of an isolated system not in equilibrium will tend to increase over time, approaching a maximum value at equilibrium.
 - **Third law of thermodynamics** – As temperature approaches absolute zero, the entropy of a system approaches a constant minimum.



Temperature v Heat

- What is the difference?
 - Although both beakers below have the same temperature ($100\text{ }^{\circ}\text{C}$), the beaker on the right has twice the amount of heat, because it has twice the amount of water.





Temperature Scales

- Measuring temperature has always been a problem.
 - The material the thermometer is made from affects the temperature reading.
 - Since different materials respond to changes in temperature differently.
 - Different scales of temperature exist ($^{\circ}\text{C}$, $^{\circ}\text{F}$).
- In 1947, the SI unit of the Kelvin was introduced as the unit for the **Absolute Thermodynamic Scale of Temperature**.

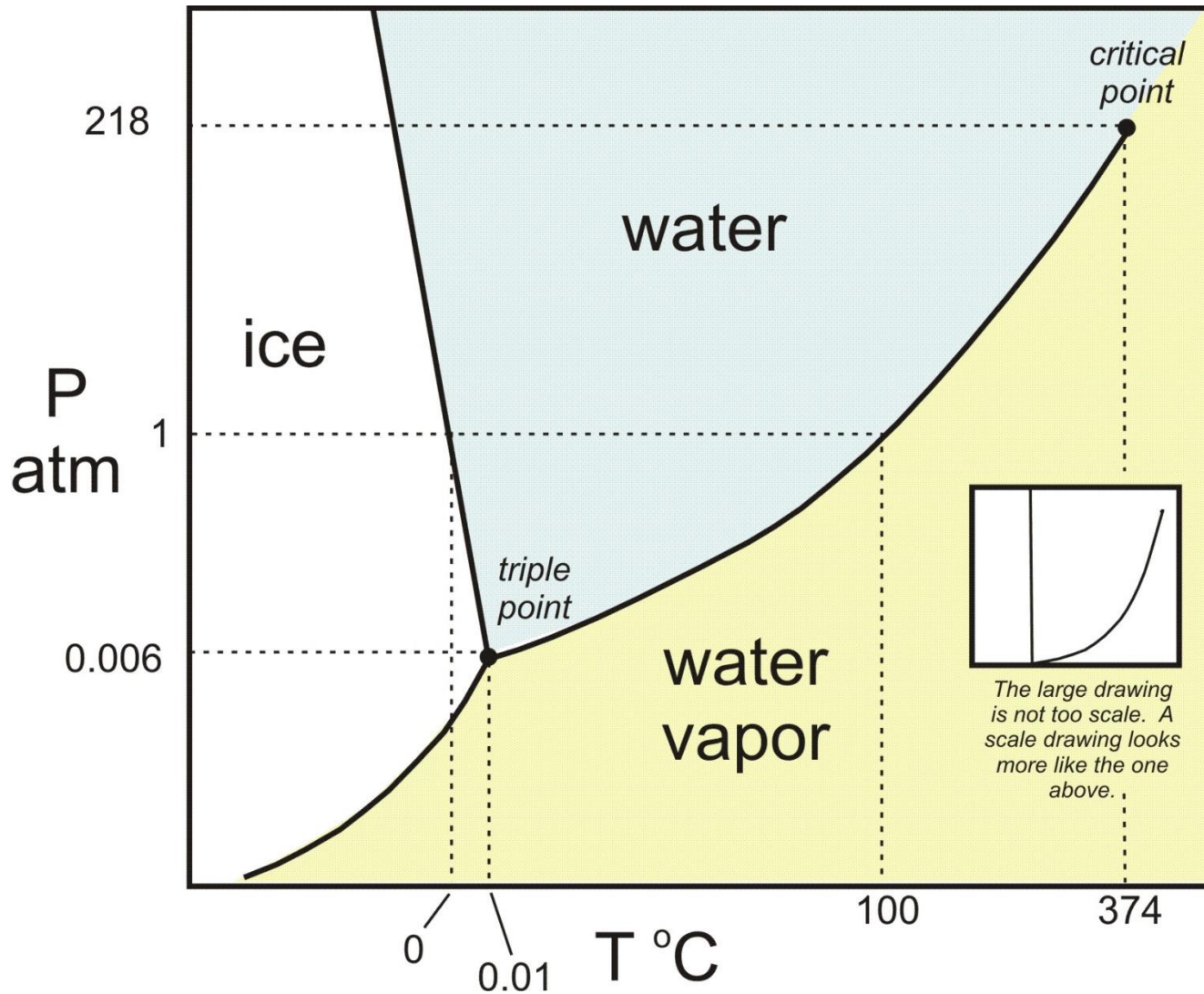


Kelvin Scale

- An absolute temperature scale.
 - Does not depend on any property of any substance.
- Starts at zero (absolute zero).
 - This is the theoretical temperature at which a substance has minimal internal energy.
- Is aligned to the Celsius scale to ease understanding.
- Uses the Triple Point of water as a reference point.



Triple Point of Water

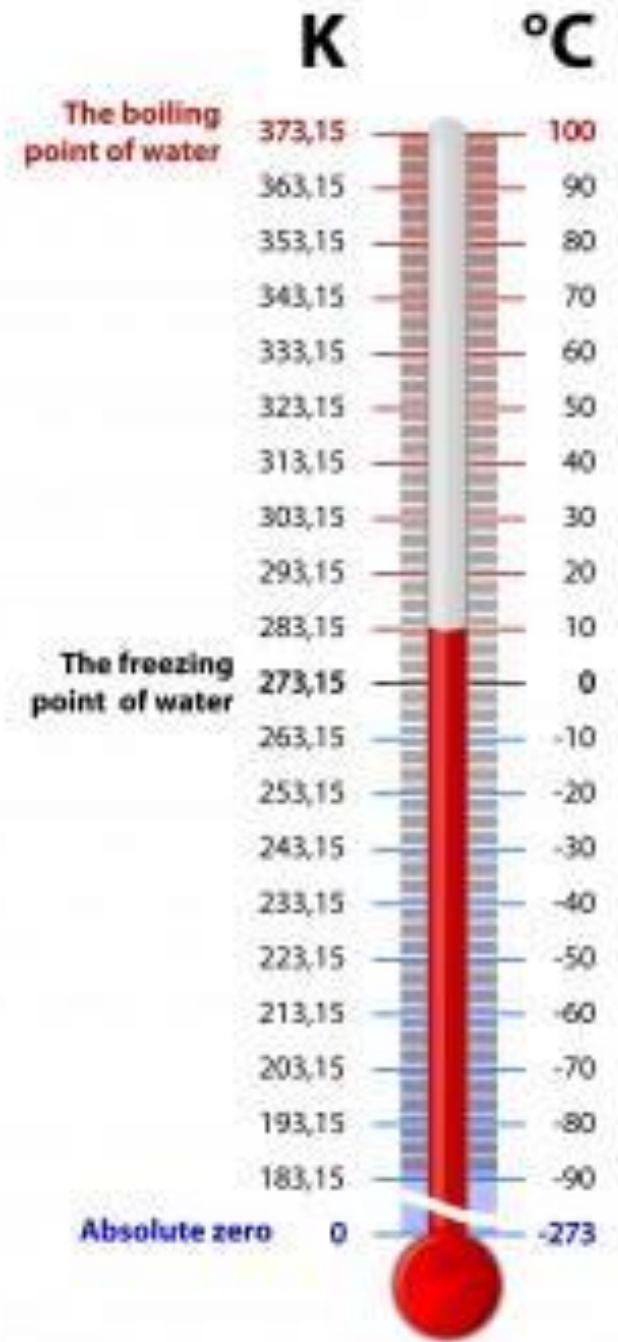


A specific temperature & pressure at which all 3 phases of a substance exist in thermal equilibrium.

Absolute scale v Celsius scale

	Absolute Scale /k	Celsius Scale /°C
Absolute zero	0	-273.15
Triple point H ₂ O	273.16	0.01
Ice point	273.15	0.00
Steam point	373.15	100.00
Room Temperature	293.15	20.00

$$T / K = t / ^\circ C + 273.15$$





5.1.1 Temperature (review)

5.1.1 Temperature

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5.1.2 Solid, Liquid & Gas

5.1.2 Solid, liquid and gas

Learning outcomes

Learners should be able to demonstrate and apply their knowledge and understanding of:

- (a) solids, liquids and gases in terms of the spacing, ordering and motion of atoms or molecules
- (b) simple kinetic model for solids, liquids and gases
- (c) Brownian motion in terms of the kinetic model of matter and a simple demonstration using smoke particles suspended in air
- (d) internal energy as the sum of the random distribution of kinetic and potential energies associated with the molecules of a system
- (e) absolute zero (0 K) as the lowest limit for temperature; the temperature at which a substance has minimum internal energy
- (f) increase in the internal energy of a body as its temperature rises
- (g) changes in the internal energy of a substance during change of phase; constant temperature during change of phase.



What is the
difference between
the density of
solids, liquids &
gases?



Solid, liquid & gas

- The density of the solid, liquid and gas phases of a substance varies greatly.

Substance	Solid density/kg m ⁻³	Liquid density/kg m ⁻³	Gas density/kg m ⁻³
oxygen	1300	1100	1.43 at STP
carbon dioxide	1600		1.98 at STP
water	920	1000	0.59 at boiling point
sodium	970	930	(0.24) at boiling point
aluminium	2700	2400	(0.12) at boiling point

STP means
Standard
Temperature
and Pressure
(273k and
100kPa (1atm))

What general conclusions can you reach?

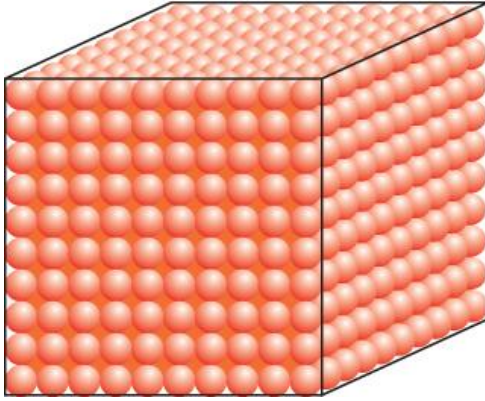


Conclusions:

- Densities of solids & liquids are of the order of 10^3 times the density of gases.
- Solids are usually (but not always) denser than liquids.
- Gas densities are very dependent on temperature & pressure. Solid/liquid densities are less so.

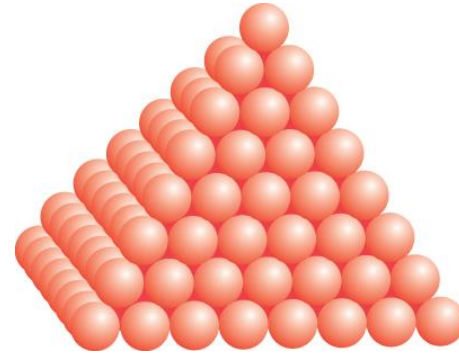


Particle Ordering - Solids



Many solids pack like this to form regular crystalline shapes

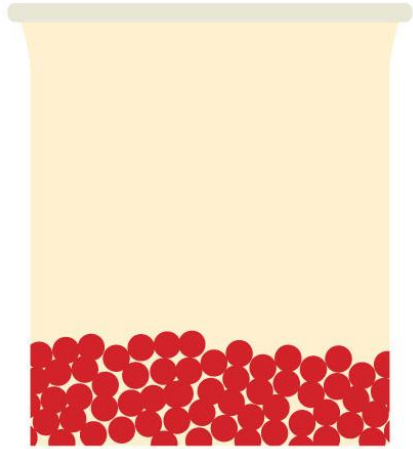
Most pack like this – with more particles per unit volume



Whichever way solids are packed, their particles are not free to move about – they are locked in position.

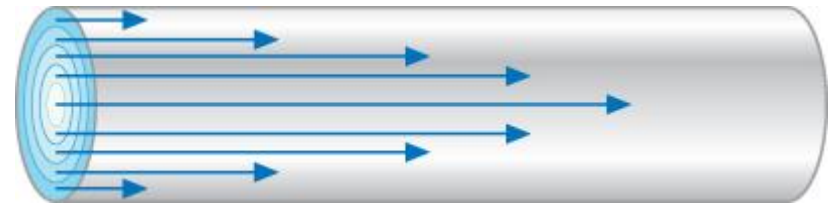


Particle Ordering - Liquids



Liquid particles are in contact but are able to move around each other

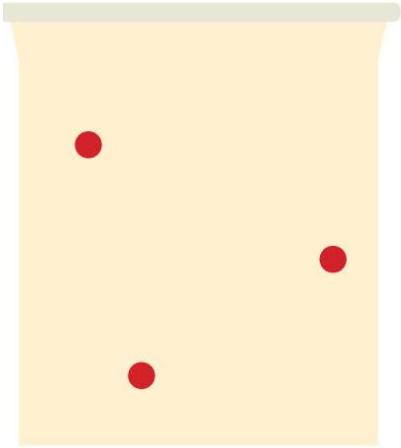
Water molecules near the edge of a pipe tend to move slower than those at the centre.



Liquid water is unusual in that it is denser than solid water.



Particle Ordering - Gases

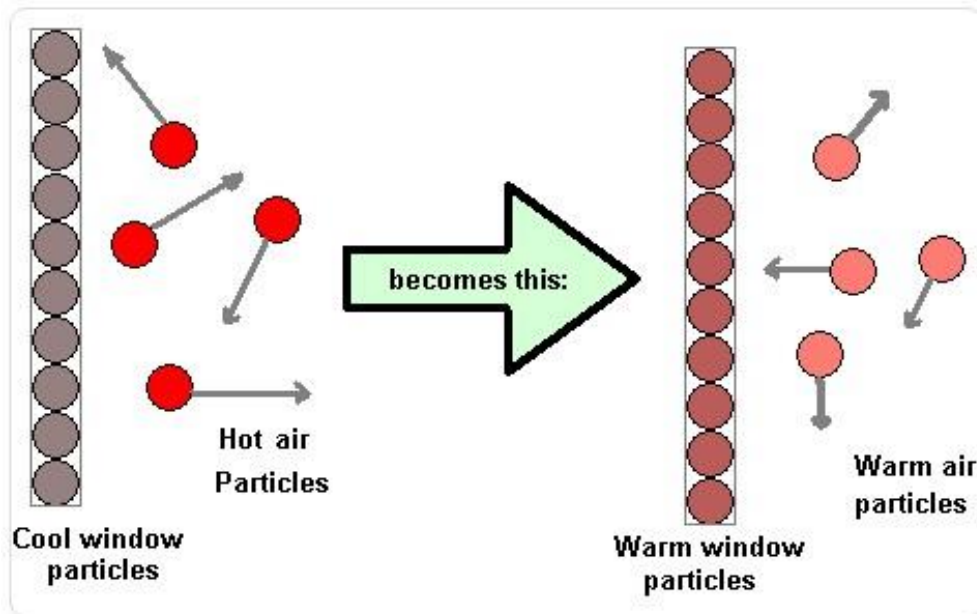


Gas particles are chaotic. They have no arrangement. They are free to move around each other and spread throughout a container.



Molecular Speeds

- Above zero Kelvin, molecules are in a state of motion (even if only oscillating about a fixed position).
 - However, they do observe the law of **conservation of energy**.



When particles collide, those with high KE lose KE and those with low KE will gain KE. The total energy in the system remains constant.



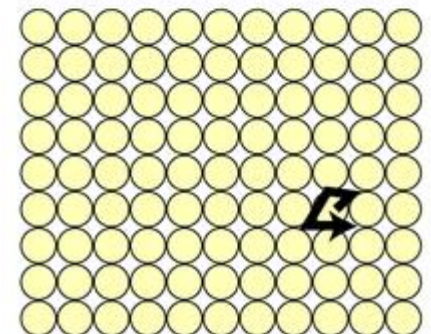
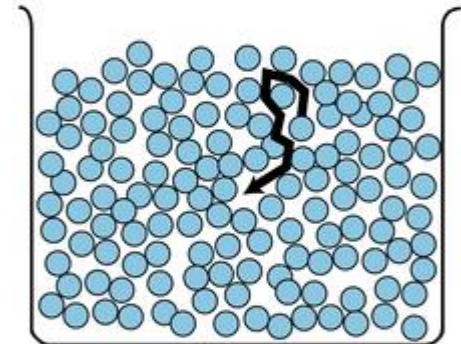
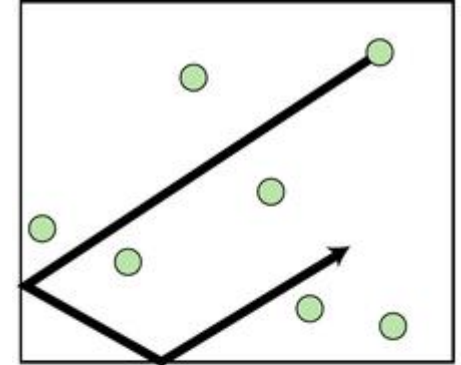
Temperature & KE

- Temperature is proportional to Kinetic Energy
 - More on this later.
- Melting point:
 - The temperature at which both solid and liquid phases can coexist.
 - Molecules of ice at 0°C have the same average kinetic energy as molecules of water at 0°C .
- The same idea holds for boiling point.



Particle Movements

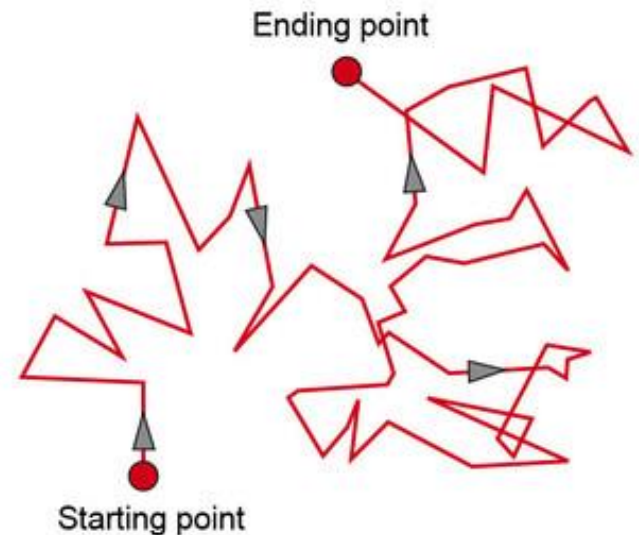
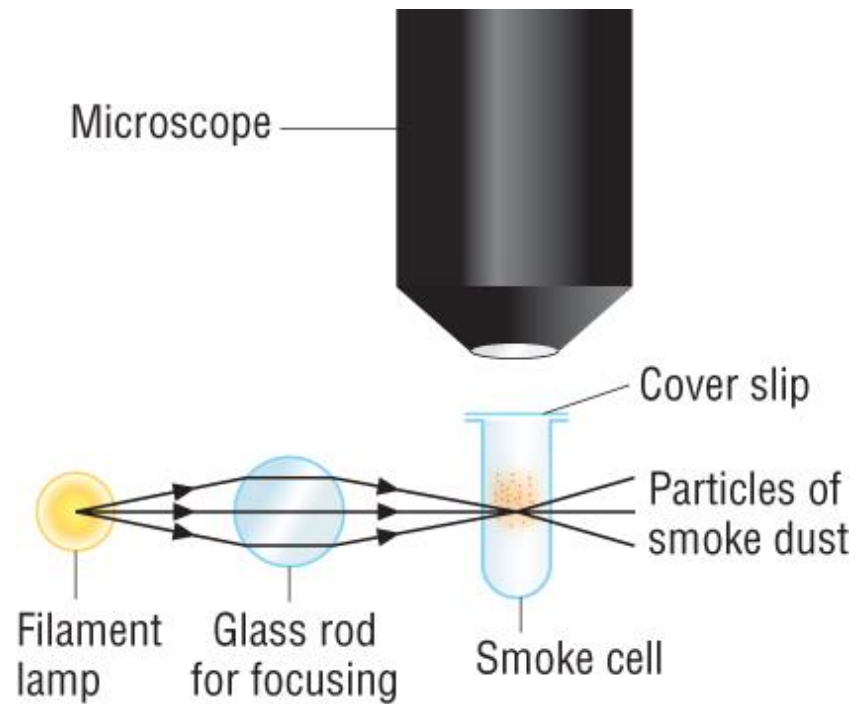
- When a solid is heated:
 - Temperature rises.
 - Kinetic energy of particles increases.
 - Greater oscillation/rotation of particles about equilibrium position.
- When a liquid is heated:
 - Same effects as above.
 - Some kinetic energy causes translational movement of particles.
- When a gas is heated:
 - Almost all of the increased kinetic energy is translational.





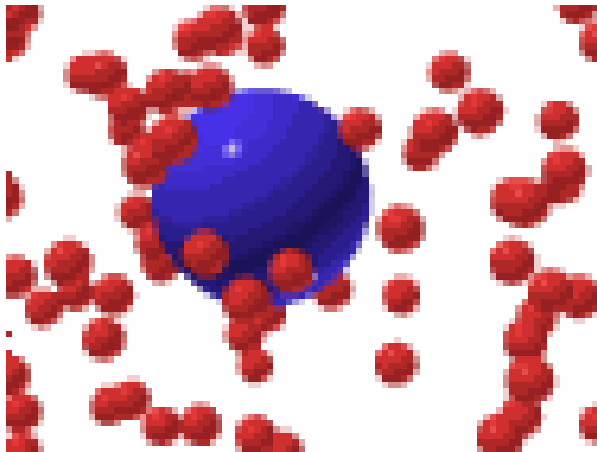
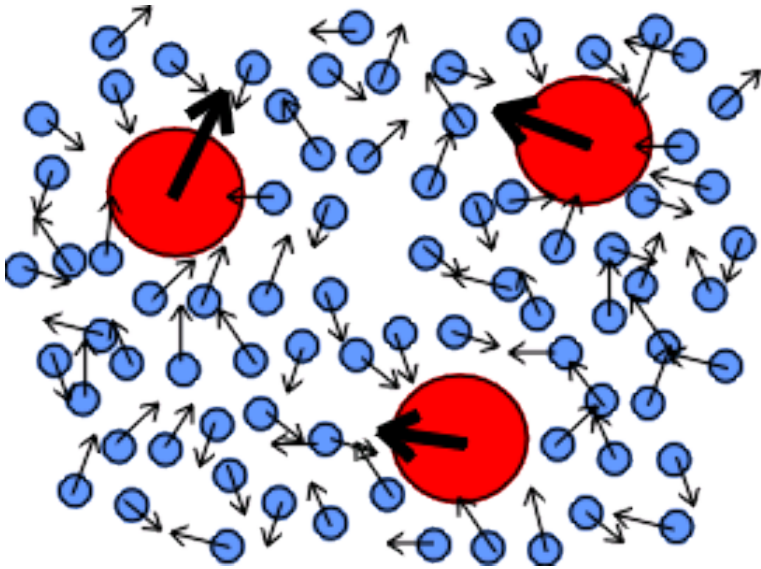
Brownian Motion

- Thomas Brown, 1827, noticed how pollen grains on the surface of water never stay still – they randomly move around.
- This can be seen with smoke particles in air, or milk powder on coffee.





An explanation



- Molecules of air are in constant motion.
- They randomly collide with the large smoke particle from all directions.
- The impulse gained by the smoke particle is not zero, resulting in an acceleration.
- If the smoke particle is too large, the impulses average to zero and the large mass results in lower acceleration.



Internal Energy

- Internal energy is the sum of the random distributions of kinetic and potential energies of all molecules in a substance.

Does a glass of water sitting on a table have any energy?

No apparent energy of the glass of water on a macroscopic scale.



Microscopic kinetic energy is part of internal energy.

Molecular attractive forces are associated with potential energy



Kinetic Energy of Molecules

- Is the sum of the random kinetic energies of all molecules within the substance.
 - The sum of all $mv^2/2$ of all molecules inside the container.
 - Does not include any additional kinetic energy gained if the container is travelling inside a vehicle.
 - Hence the term **RANDOM** kinetic energy.



Calculating Kinetic energy of Gases

- Random KE = $\frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2 + \frac{1}{2}mv_3^2 + \dots + \frac{1}{2}mv_n^2$.
 - Where n is the number of molecules in the gas and m is the mass of each gas molecule.
- This can be rewritten as $\frac{1}{2}m(v_1^2 + v_2^2 + v_3^2 + \dots + v_n^2)$.
- The term in brackets is equal to the mean square speed of all molecules multiplied by the number of molecules.
- So the expression becomes:

$$KE = \frac{1}{2}m\overline{v^2}n$$

And since mn is the total mass of all molecules, which is equal to volume x density:

$$KE = \frac{1}{2}V\rho\overline{v^2}$$



Potential Energy of Molecules

- Is the sum of all random potential energies resulting from electrostatic intermolecular forces between the molecules.
 - Does not include any potential energy changes due to raising or lowering the container.
 - We do not need to calculate these!
 - (pew)



Factors Affecting Internal Energy

- **Temperature**
 - With high temperature comes more rapid molecular movement, leading to higher kinetic energies and therefore internal energy.
- **Pressure**
 - In gases, molecules attract each other so when a gas expands (or pressure is reduced) work will be done to separate the molecules, leading to a higher potential energy and thus higher internal energy.
- **State**
 - The state of a substance describes its phase (gas, liquid, solid) and also things like atomic arrangements or crystalline structure.
 - A change in state does not involve a change in temperature so the total kinetic energy does not change.
 - However, a change in state does involve the breaking or forming of molecular attractions.
 - In gases:
 - Electrostatic potential energy is zero, the electrostatic forces between molecules is negligible since they are so far apart.
 - In liquids:
 - Electrostatic potential energy is negative – energy must be supplied to break the intermolecular bonds.
 - In solids:
 - Intermolecular electrostatic forces are very large resulting in a very large negative electrostatic potential energy.



Factors which have no effect on internal energy

- **Speed**

- The speed of the whole gas container does not affect the random kinetic energies of the molecules
 - So internal energy remains constant.

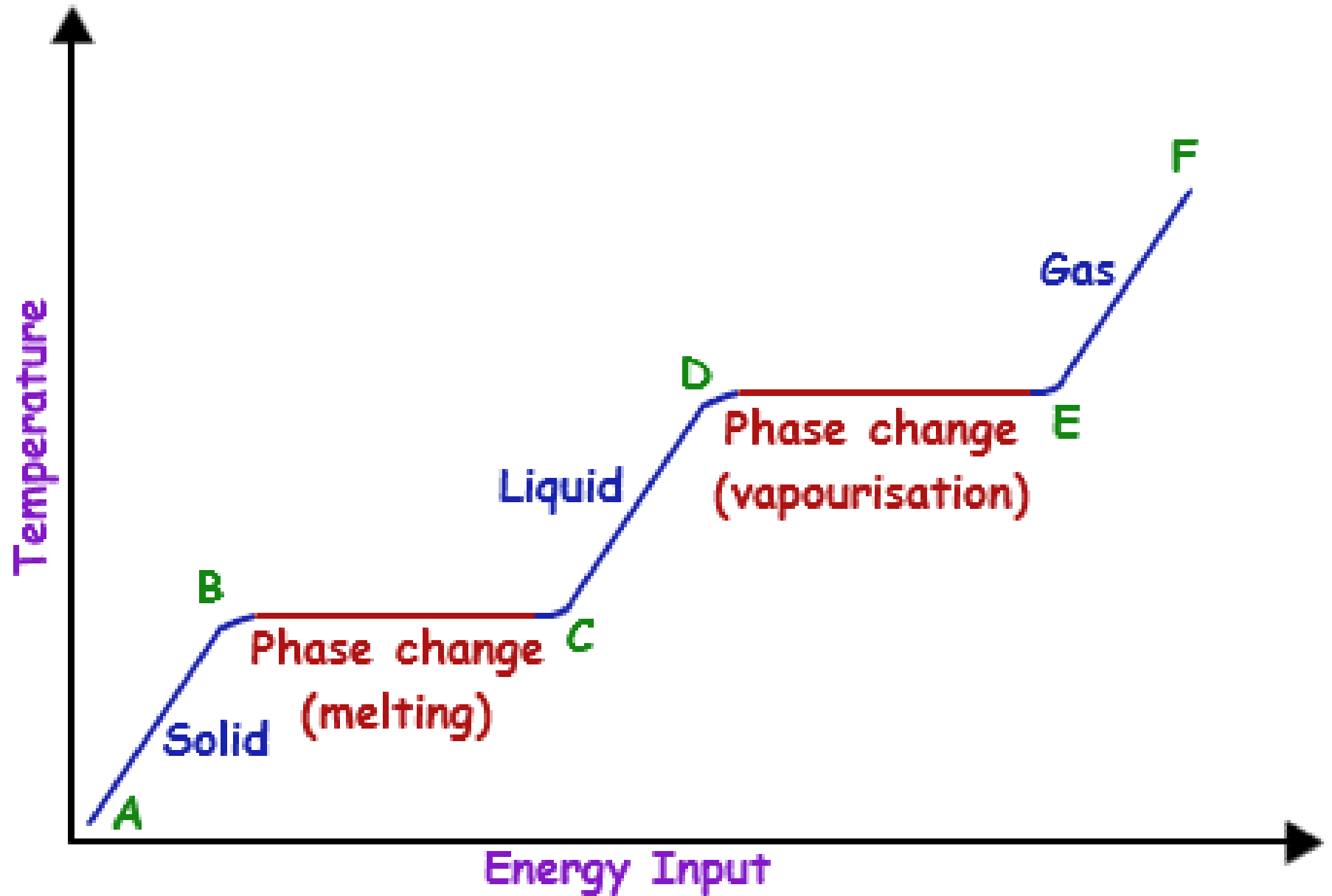
- **Height**

- The height of the whole gas container in a gravitational field does not affect the random potential energies of the molecules.
 - So internal energy remains constant.



Practical

- Plot a temperature v time graph for melting then boiling ice in a beaker over a bunsen.
- Think about:
 - How you can ensure the heat energy supplied is constant.
 - How you can accurately measure the temperature of the ice/water rather than the beaker itself.
- Write:
 - On the graph, highlight the parts where:
 - Internal energy is increasing.
 - Potential energy is increasing.
 - Kinetic energy is increasing.





Practical 2

- Wipe some acetone on the back of your hand.
 - What does it feel like when the acetone evaporates?
 - In terms of internal, kinetic and potential energies explain why you feel this sensation.



5.1.2 Solid, Liquid & Gas (review)

5.1.2 Solid, liquid and gas

Learning outcomes

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- (f) increase in the internal energy of a body as its temperature rises
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5.1.3

Thermal Properties of Materials

5.1.3 Thermal properties of materials

Learning outcomes

Learners should be able to demonstrate and apply their knowledge and understanding of:

- (a) specific heat capacity of a substance; the equation $E = mc\Delta\theta$
- (b)
 - (i) an electrical experiment to determine the specific heat capacity of a metal or a liquid
 - (ii) techniques and procedures used for an electrical method to determine the specific heat capacity of a metal block and a liquid
- (c) specific latent heat of fusion and specific latent heat of vaporisation; $E = mL$
- (d)
 - (i) an electrical experiment to determine the specific latent heat of fusion and vaporisation
 - (ii) techniques and procedures used for an electrical method to determine the specific latent heat of a solid and a liquid.



What, specifically,
is Specific Heat
Capacity?



Specific Heat Capacity, c

- SHC = the quantity of thermal energy required to raise the temperature of a unit mass of a substance by a unit temperature rise.



Some definitions:

- Specific = per unit mass.
- Heat = energy, measured in joules.
 - The term “heat” is only used because it has been for years.
 - We tend not to use “heat” to mean energy any more.
 - We use more descriptive terms instead such as internal energy.
 - We use “heat” as a verb or “heating” as a process whereby energy is transferred from one object to another.



SHC equation

$$c = \frac{E}{m\Delta\theta} \quad \text{or} \quad E = mc\Delta\theta$$

- Where:

E = thermal energy (energy supplied to an object).

Measured in joules

m = the mass of the material being heated.

Measured in kilograms

$\Delta\theta$ = the change in temperature.

Measured in °C or K

NOTE: a change in temperature is the same whether °C or K is used.



Measuring SHC – Practical 1

- Measure SHC of water using a kettle.
 - Add a known mass and known temperature of water to an empty kettle.
 - Switch the kettle on and measure the time taken for the water to reach 60°C .
 - Find the power of the kettle from its base.
 - Use Pt to calculate energy transferred.
 - Calculate c and compare with published value for water ($4190\text{Jkg}^{-1}\text{K}^{-1}$).
 - Why are the two values different?



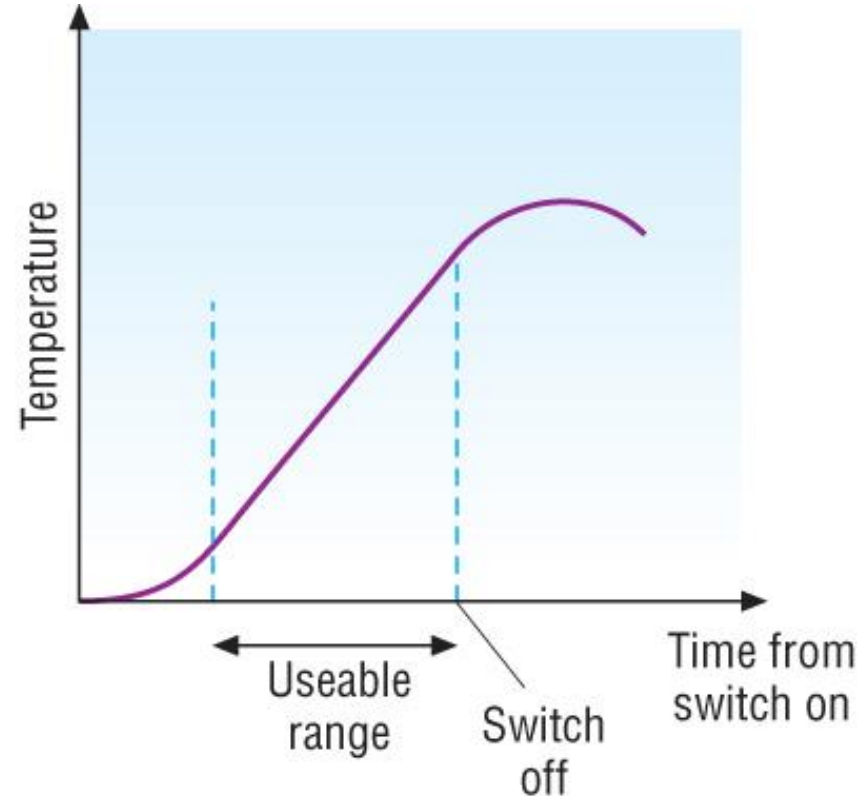
Measuring SHC - Practical 2

- Measure SHC of aluminium using an electrical method.



Using a Temperature-Time graph

- Plot temperature against time for the aluminium block.
 - How can this be used to calculate SHC?
- Over the useable range, the gradient = $\Delta\theta/t$



$$c = \frac{E}{m\Delta\theta} = \frac{VI t}{m\Delta\theta} = \frac{VI}{m} \times \frac{1}{\text{gradient}} = \frac{P}{m \times \text{gradient}}$$



Measuring SHC - Practical 3

- Measure SHC of lead shot using GPE to heat the metal.



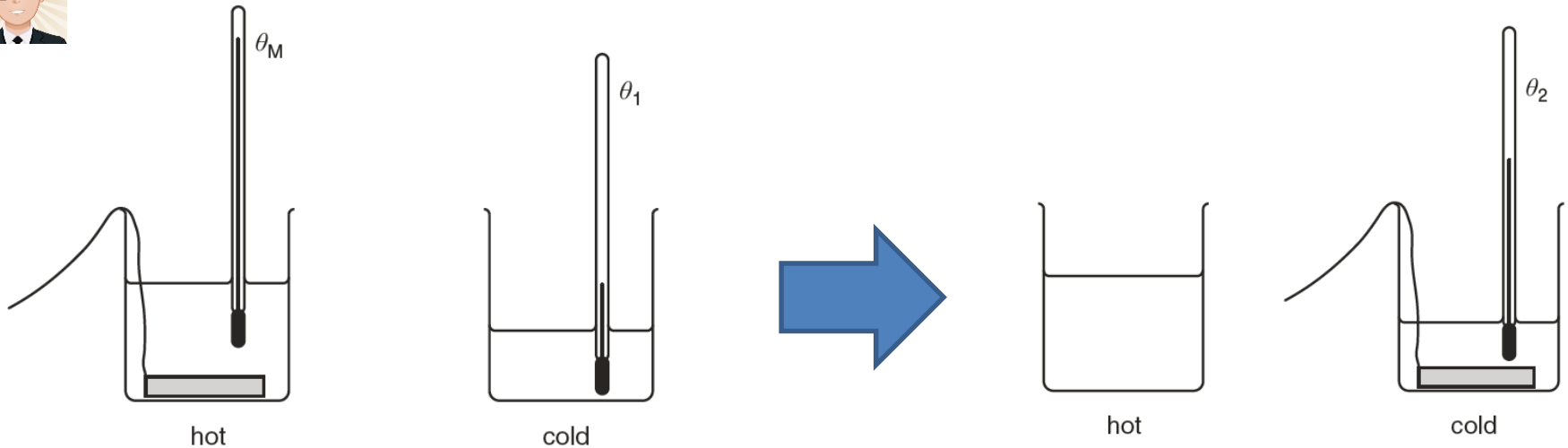
Method of Mixtures

- Two substances at different known temperatures are mixed together until they reach thermal equilibrium.
- If we know the final temperature of the mixture and the SHC of one of the two substances we can calculate the SHC of the other substance.



Measuring SHC – Practical 4

- Use the Method of Mixtures to measure the SHC of an aluminium 1kg mass, when the published SHC of water is: $4.2\text{Jkg}^{-1}\text{K}^{-1}$.



The thermal energy transferred from the heated metal to the cold water is equal to the thermal energy gained by the cold water from the heated block.

$$m_{metal} c_{metal} \Delta \theta_{metal} = E = m_{water} c_{water} \Delta \theta_{water}$$

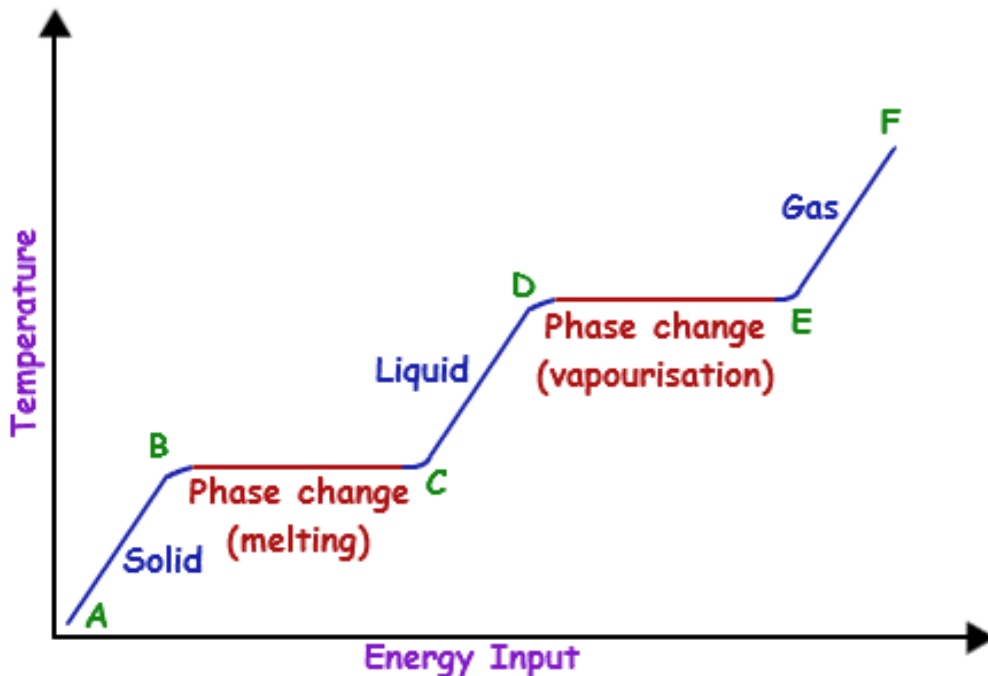
Simply rearrange the equation to find the SHC of the aluminium block:

$$c_{metal} = \frac{m_{water} c_{water} \Delta \theta_{water}}{m_{metal} \Delta \theta_{metal}}$$



Phase Changes

- As a substance changes from solid to liquid (melts) or from liquid to gas (boils), two phases are in existence at the same time.



B-C and D-E:

Temperature is constant, so internal kinetic energy is constant. Yet total internal energy is increasing so internal potential energy is increasing.



Latent Heat (hidden heat)

- **Specific Latent Heat of Fusion, L_f :**

- The quantity of energy per unit mass of substance required to change it from solid to liquid at a constant temperature.

$$E = mL_f$$

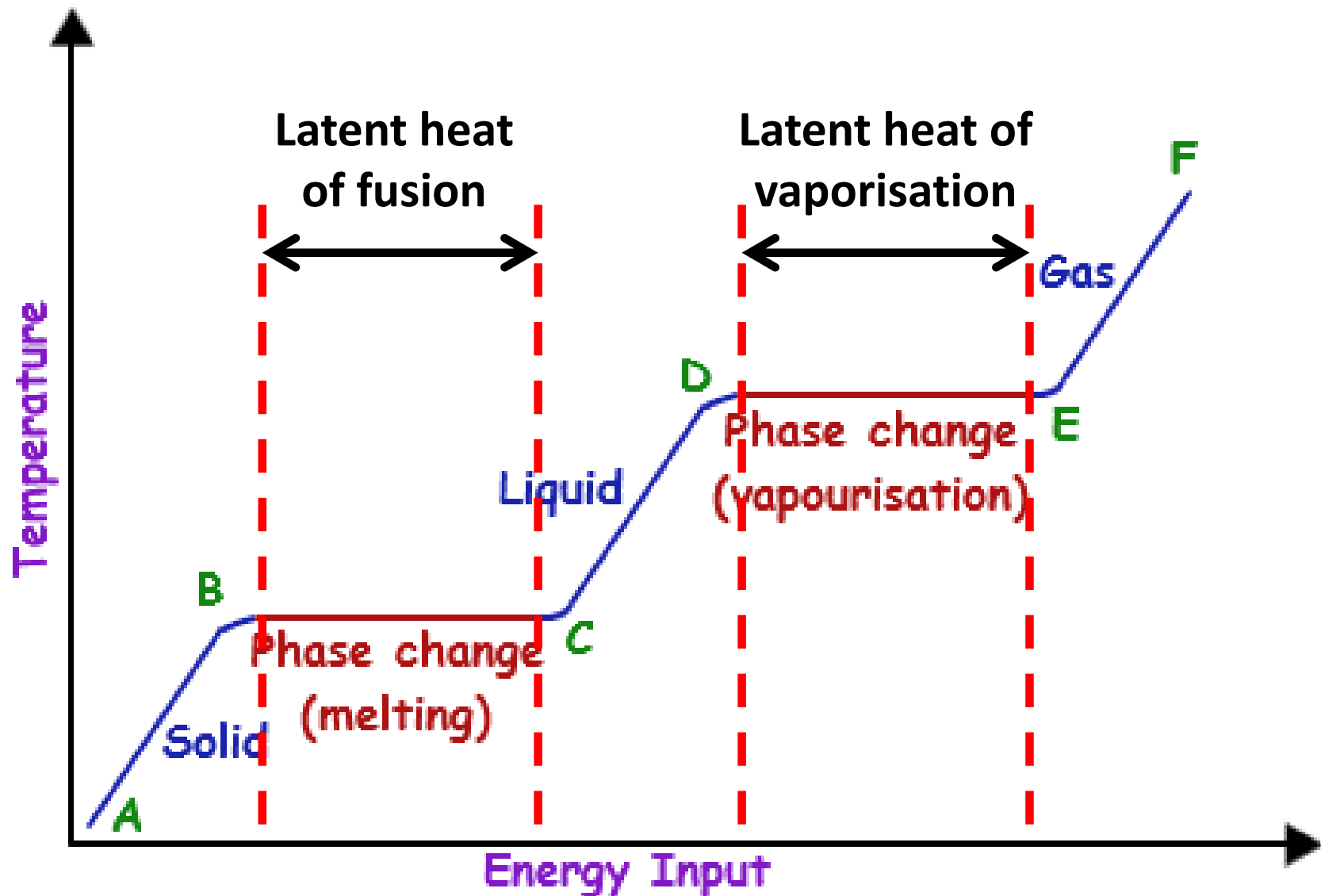
- **Specific Latent Heat of Vaporisation, L_v :**

- The quantity of energy per unit mass of substance required to change it from liquid to vapour at a constant temperature.

$$E = mL_v$$

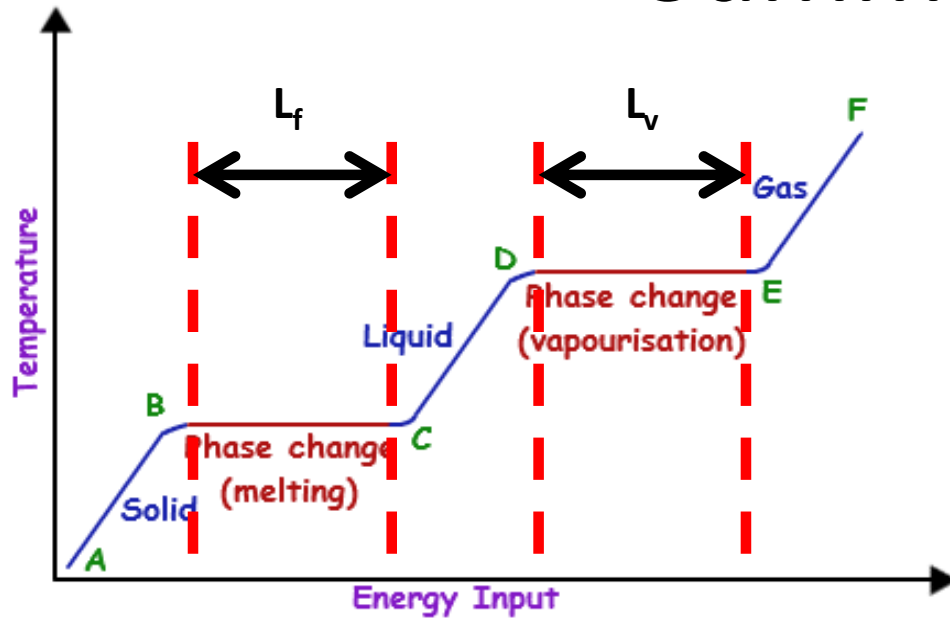


Diagrammatically:





Summarise:



For each section
(A-B, B-C, C-D, D-E & A-F):

Write an algebraic
expression for the energy
added to the substance.

A-B: $E = mc_{solid}\Delta\theta$

B-C: $E = mL_f$

C-D: $E = mc_{liquid}\Delta\theta$

D-E: $E = mL_v$

A-F: $E = mc_{solid}\Delta\theta + mL_f + mc_{liquid}\Delta\theta + mL_v + mc_{gas}\Delta\theta$



5.1.3

Thermal Properties of Materials (review)

5.1.3 Thermal properties of materials

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5.1.4 Ideal Gases

5.1.4 Ideal gases

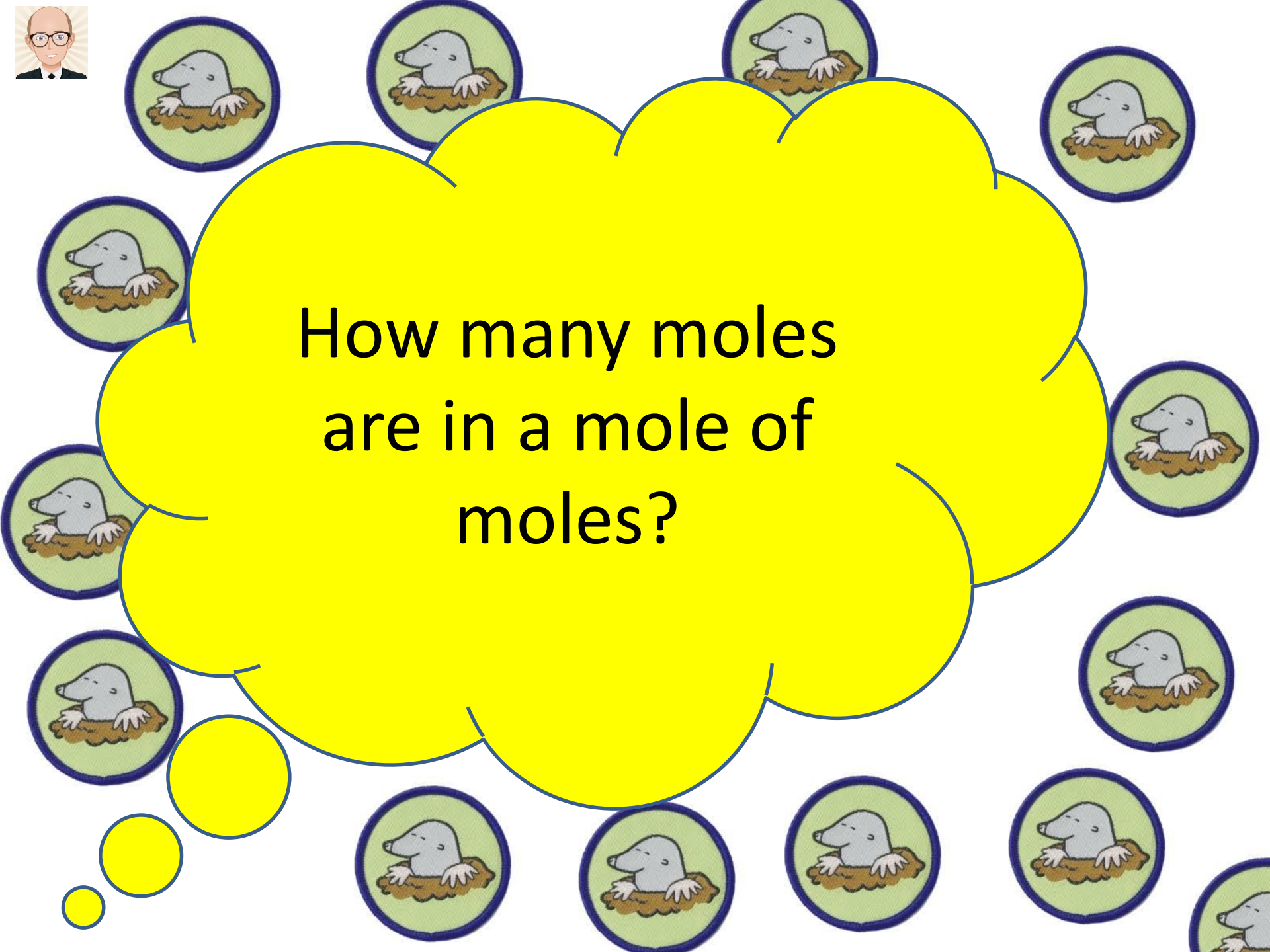
Learning outcomes

Learners should be able to demonstrate and apply their knowledge and understanding of:

- (a) amount of substance in moles; Avogadro constant N_A equals $6.02 \times 10^{23} \text{ mol}^{-1}$
- (b) model of kinetic theory of gases
- (c) pressure in terms of this model
- (d)
 - (i) the equation of state of an ideal gas $pV = nRT$, where n is the number of moles
 - (ii) techniques and procedures used to investigate $PV = \text{constant}$ (Boyle's law) and $\frac{P}{T} = \text{constant}$
 - (iii) an estimation of absolute zero using variation of gas temperature with pressure
- (e) the equation $pV = \frac{1}{3}Nm\overline{c^2}$, where N is the number of particles (atoms or molecules) and $\overline{c^2}$ is the mean square speed
- (f) root mean square (r.m.s.) speed; mean square speed
- (g) the Boltzmann constant; $k = \frac{R}{N_A}$
- (h) $pV = NkT$; $\frac{1}{2}m\overline{c^2} = \frac{3}{2}kT$
- (i) internal energy of an ideal gas.



How many moles
are in a mole of
moles?





The Mole



- Words that represent numbers:
 - Couple:
 - TWO
 - Dozen:
 - TWELVE
 - Mole:
 - 600,000,000,000,000,000,000,000
- 6×10^{23} is **Avagadro's constant**, N_A .
 - This is the number of atoms contained in 12g of Carbon-12



The mass of one mole of atoms equals the relative atomic mass in grams

- One mole of Carbon atoms has a mass of 12.0g.
- One mole of Calcium atoms has a mass of 40.1g.
- One mole of Nitrogen atoms has a mass of 14.0g.

- Be careful though:
 - One mole of Hydrogen gas has a mass of 2.0g.
 - One mole of Nitrogen gas has a mass of 28.0g.



Molar Mass, M , and Amount of Substance, n

Molar Mass, M

- The mass of one mole of a substance.
- Has the units of g mol^{-1}
 - Eg: $M(\text{CO}_2) = 12.0 + (16.0 \times 2) = 44 \text{ g mol}^{-1}$.

$$\text{Amount, } n = \frac{\text{Mass, } m}{\text{Molar Mass, } M}$$

Amount of Substance, n

- The number of moles of particles in a substance.
- Has the unit of mol.

$$n = \frac{m}{M}$$



What's so great
about an Ideal Gas?



Studying gases

- Gases can be described either on a macroscopic level or at a microscopic level:
 - Macroscopic descriptions
 - Volume, Temperature, Pressure
 - Microscopic descriptions
 - Particle velocities, momenta, forces



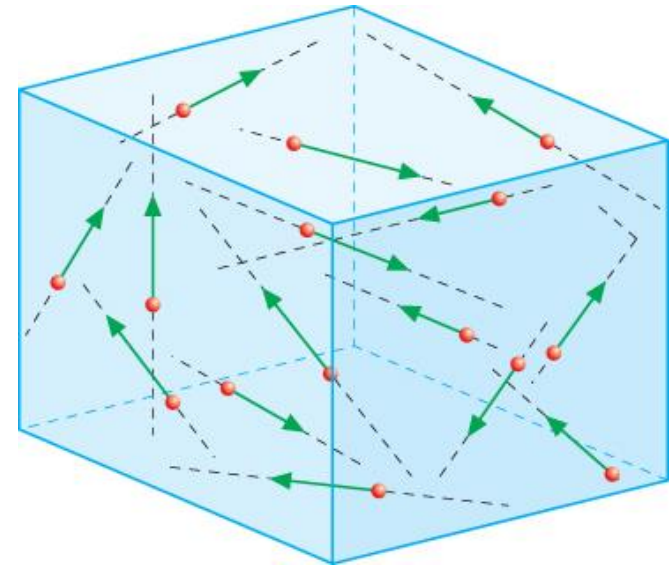
Kinetic Gas Model

- A gas is a phase of matter with mass and a volume equal to that of its container.
 - A gas will exert a pressure on the walls of its container from the collisions of molecules with these walls.
- A kinetic model of a gas describes the motion of its molecules.
 - We can also calculate the pressure exerted by a gas if we make some simplifying assumptions:



Kinetic Gas Model Assumptions

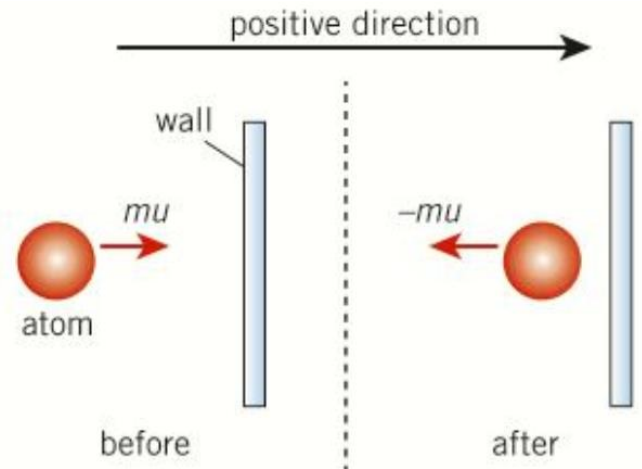
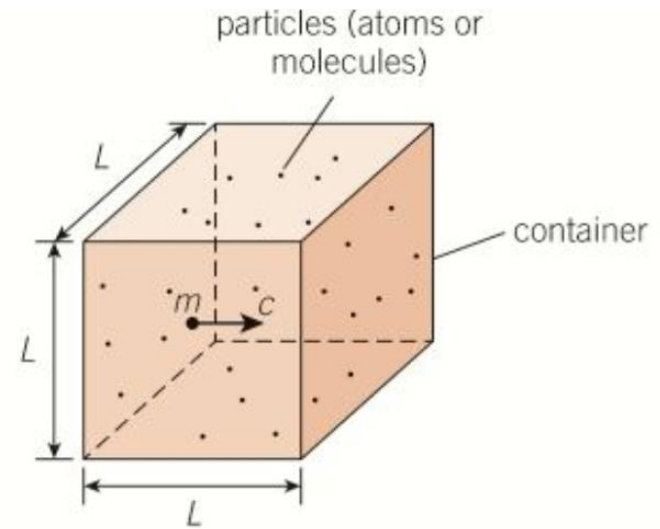
- Gas molecules:
 - Move rapidly, in random directions.
 - Collide with each other and with the container walls.
 - These collisions are elastic (no loss of kinetic energy).
 - Experience a negligible gravitational force.
 - Experience no intermolecular forces except during collisions.
 - The time during a collision is negligible compared to the time between collisions.
 - Have a total volume which is negligible compared with the volume of the container, despite there being a large number of them.





Gas Pressure

- Using the ideal gas assumptions and some of Newton's Laws, we can show how pressure is created:
 - Particles in a gas are always moving in random directions (with a mass, m , and a velocity, c).
 - When they collide with the container walls they bounce off elastically.
 - The force on the wall is equal & opposite to the force on the particle.
 - This force is proportional to the change in momentum, Δp .
 - The change in momentum is equal to $-2mu$.
 - Pressure is a measure of the total force on the wall compared to the area of the wall ($p=F/A$)





Gas Theories

- **Macroscopic theories:**
 - Concerned with temperature, volume & pressure of gases.
 - Described by some simple **gas laws**.
- **Microscopic theories:**
 - Concerned with the movements of individual molecules and how they collide with each other and the walls of the container.
 - Eg the kinetic gas model described earlier and Boltzman's equations described later.



Boyle's Law

- A macroscopic approach.
- The volume of a fixed mass of an ideal gas is inversely proportional to the pressure exerted on it, provided the temperature is kept constant.

$$V \propto \frac{1}{p} \quad \text{Or} \quad pV = \text{Constant}$$

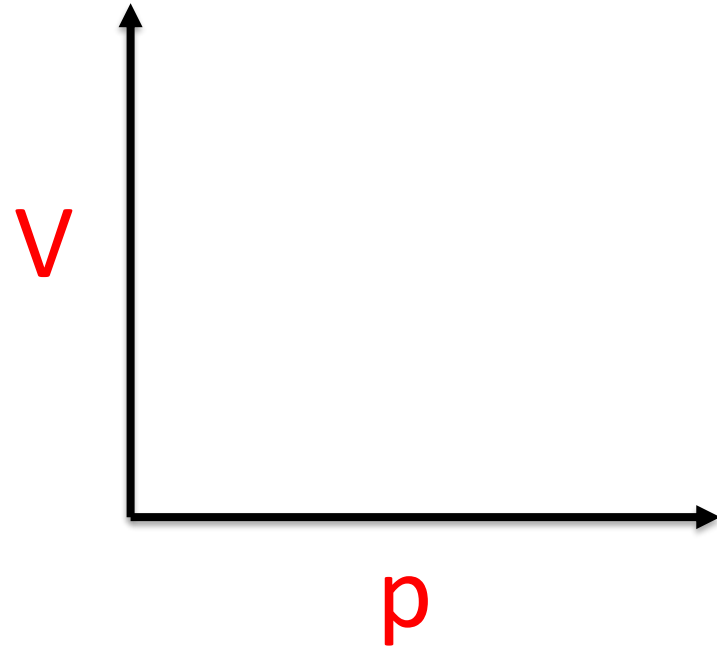


Practical

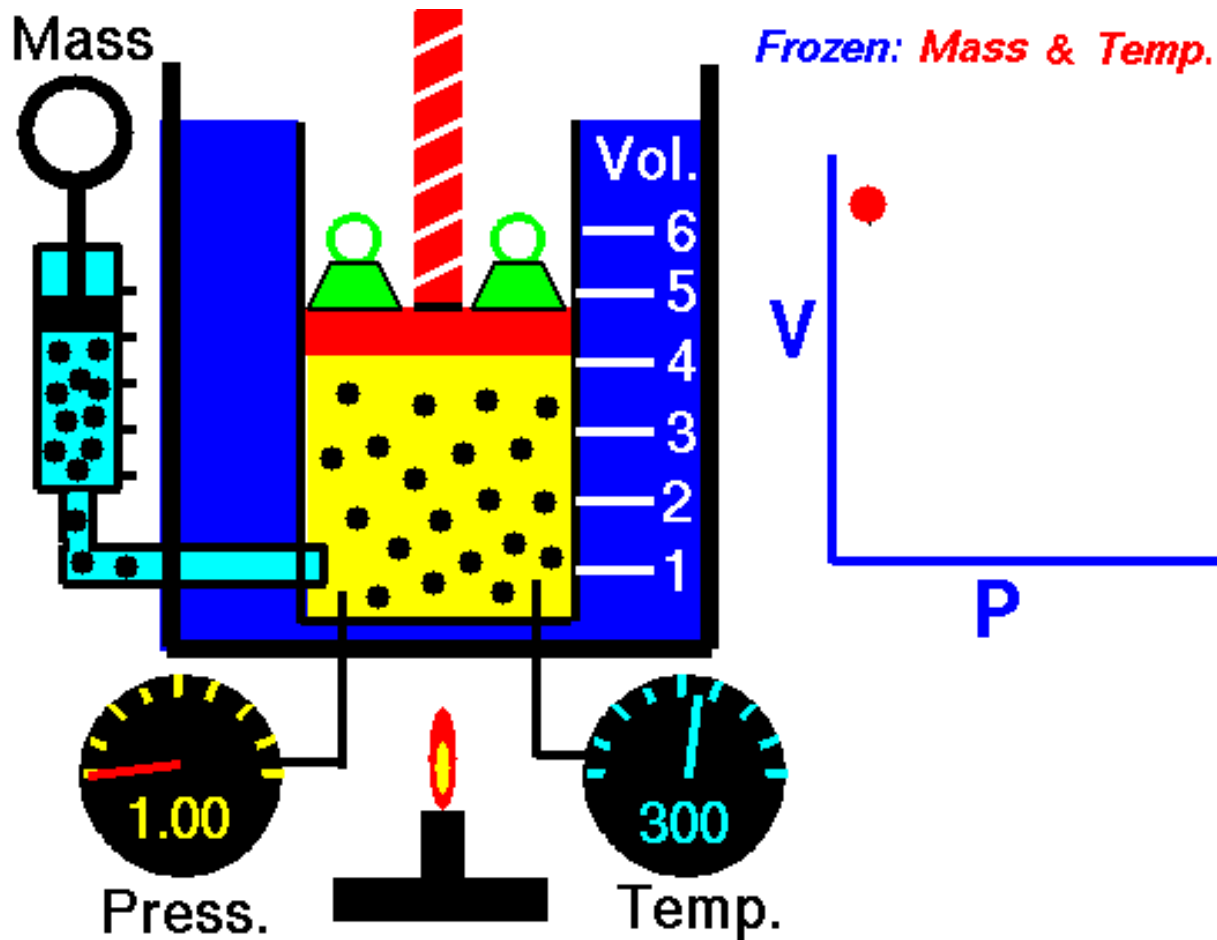
- Use the plugged syringes to plot volume against pressure for a variety of different pressures.

$$p = F / A$$

$$F = mg$$



$$V \propto \frac{1}{p} \quad \text{Or} \quad pV = \text{Constant}$$



When comparing the same gas under different conditions:

$$p_1 V_1 = p_2 V_2$$



Boyle's Law Practical



- An oil filled tube with a pressure gauge and pump can be used to demonstrate Boyle's Law.
 - Pressure can be varied using the pump.
 - Air volume can be measured on the scale.
 - Allow the temperature to equilibrate after each pressure change.
 - Plot p against $1/V$ not p against V .
 - Hard to tell an inverse relationship with a p - V graph but a p - $1/V$ graph will give a straight line if $p \propto 1/v$.
 - Try it with yours!!



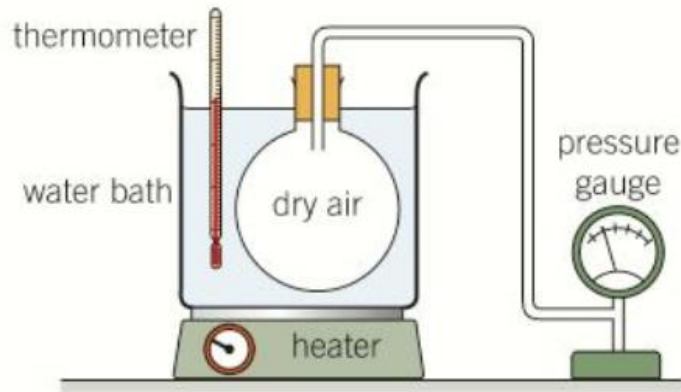
Pressure v Temperature

- Another macroscopic approach.
- The pressure of a fixed mass of an ideal gas is directly proportional to its absolute temperature, provided the volume is kept constant.

$$p \propto T \quad \text{Or} \quad \frac{p}{T} = \text{Constant}$$

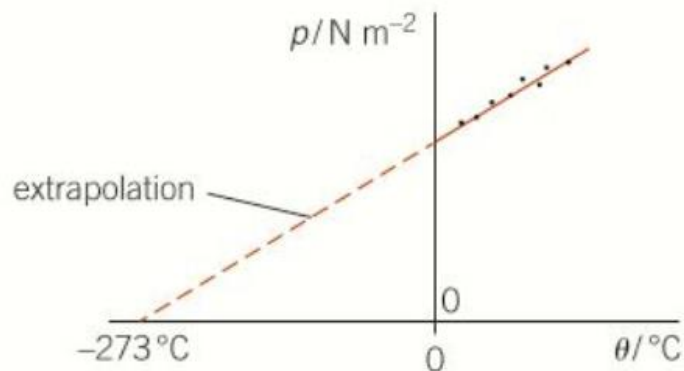


Estimating Absolute Zero



▲ **Figure 5** Apparatus used to determine absolute zero through investigating how the temperature of a gas affects its pressure

1. Set up the apparatus as shown.
2. Measure the pressure of air inside the Jolly Bulb for a variety of temperatures.
3. Plot pressure against temperature.
4. Extrapolate the line of best fit until it reaches $p=0$.
5. Read the estimated value of absolute zero.





Charles' Law

- For a fixed mass of an ideal gas at constant pressure, its volume will be proportional to the ideal gas temperature in kelvin.

$$V \propto T$$

So... $\frac{V}{T} = \text{a constant}$

And... $\frac{V_1}{T_1} = \frac{V_2}{T_2}$



Combining the gas laws:

If... $p \propto \frac{1}{V}$ and... $p \propto T$

then... $p \propto \frac{T}{V}$ therefore... $\frac{pV}{T} = \text{constant}$

so... $\frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2}$

for the same gas
at different
conditions



Moles – they bite so be careful!

- You need to ensure you understand whether you're talking about atoms or molecules.
 - “Particles” refers to either.
 - A mole of hydrogen atoms has 6.02×10^{23} atoms and has a mass of 1g.
 - A mole of hydrogen molecules has 6.02×10^{23} molecules and has a mass of 2g.



The Molar Gas Constant, R

- We've just seen how an ideal gas undergoing changes to pressure, volume or temperature has pV/T which remains constant.

$$\frac{pV}{T} = \text{constant}$$

- But what is this constant?
- This constant is equal to the number of moles of the gas, n , multiplied by the **Molar Gas Constant**, R .



What is R

So... $\frac{pV}{T} = nR$

Where:

n is the number of moles

R is the molar gas constant.

Rearranging... $pV = nRT$

This is one of the ideal gas equations.

For all gases, R is $8.3145 \text{ Jmol}^{-1}\text{K}^{-1}$.



Calculate:

- The volume of a mole of nitrogen molecules at standard pressure (101300Pa) & temperature (273.2K).

$$pV = nRT$$

Chemists will recognise this as 22.43 dm³.

$$V = \frac{nRT}{p}$$

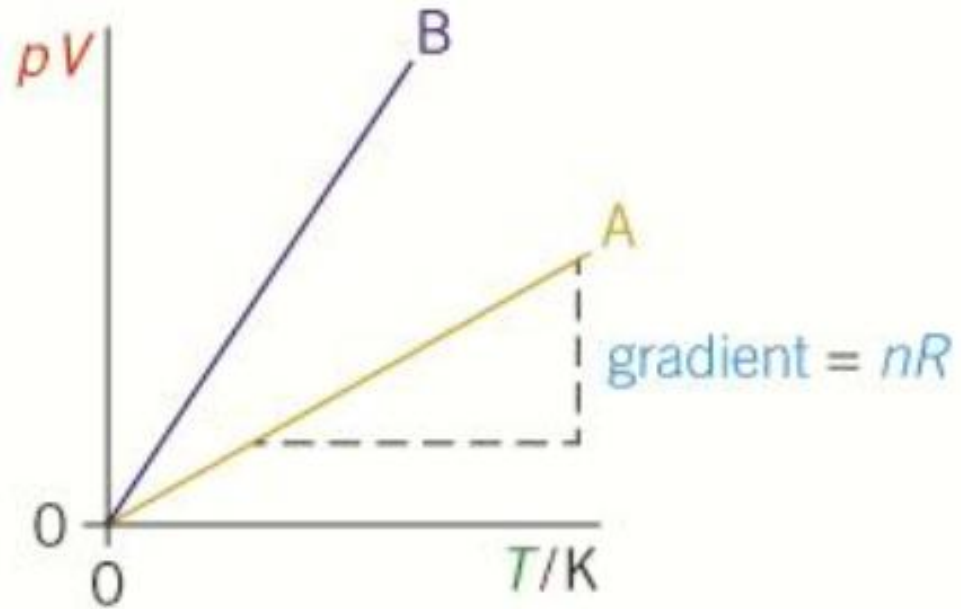
This value is the same for all gases.

$$V = \frac{1 \times 8.3145 \times 273.2}{101300} = 22.43 \times 10^{-3} m^3$$



Graphically

- We know by now how the exam boards love a good graph:



▲ **Figure 7** A graph of pV against T for a fixed amount of gas produces a straight line through the origin. Gas B produces a steeper line than gas A, as gas B contains a greater number of moles than A.



What's the average
velocity of particles
in a gas?

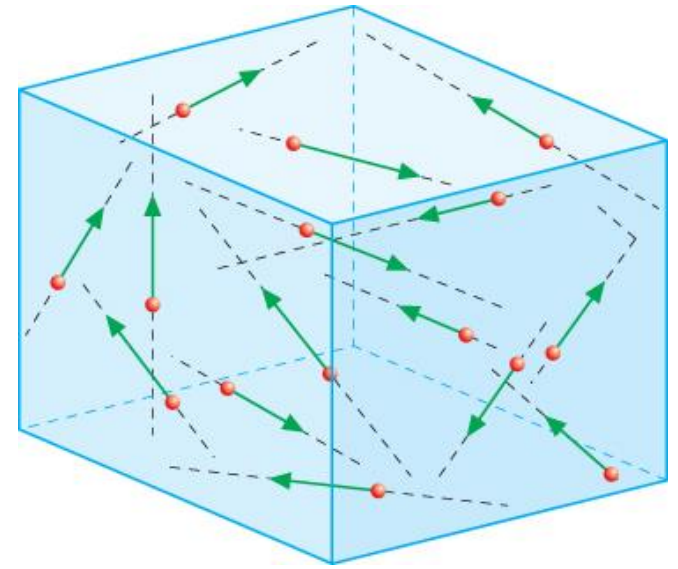


0ms^{-1}



What!!??

- Velocity is a vector.
- The velocities of all the particles in a gas will cancel out, since they are travelling in random directions.
- Instead of mean velocity we use **root mean square speed**.





Root mean square speed

- Instead of finding the mean velocity, c , of all particles (which is zero).
- We find the mean squares of velocities, $\overline{c^2}$.
 - This ensures they do not cancel out.
- We then square root this value to find the root mean square speed, $\sqrt{\overline{c^2}}$, or $c_{r.m.s.}$.



Gas Pressure (again)

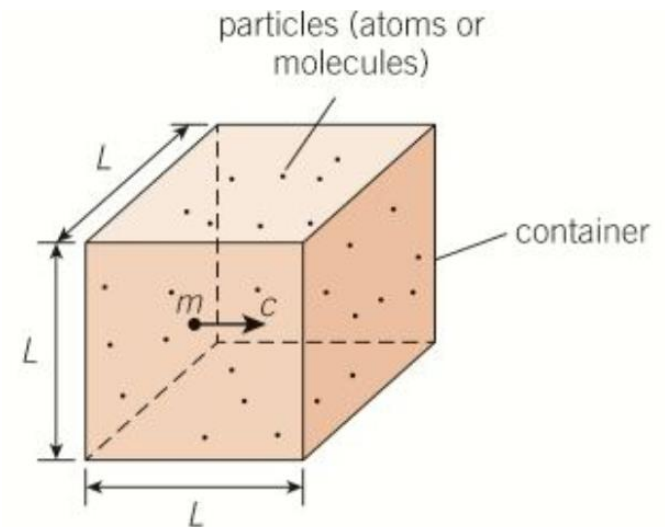
- We saw how gas pressure can be described at the macroscopic level with $p=F/A$.
- This time we look at how pressure occurs at the microscopic level.

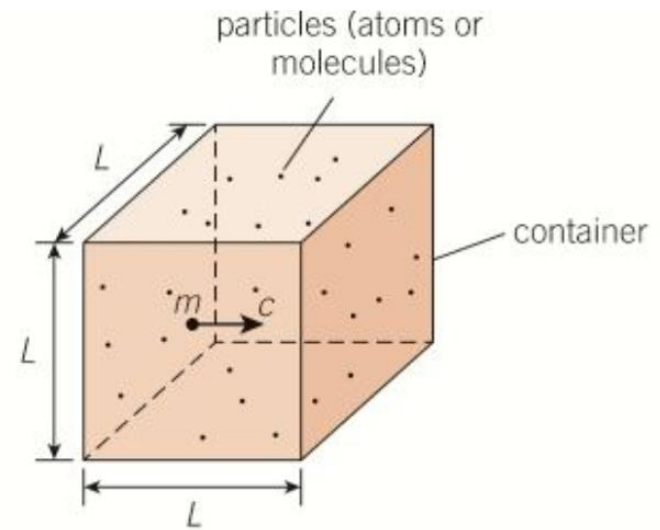


The equation for pressure & volume

$$pV = \frac{1}{3} N m \overline{c^2}$$

- You can derive this equation fairly easily:
 - Consider a single particle with mass, m , & velocity, v , in a box with sides, L .

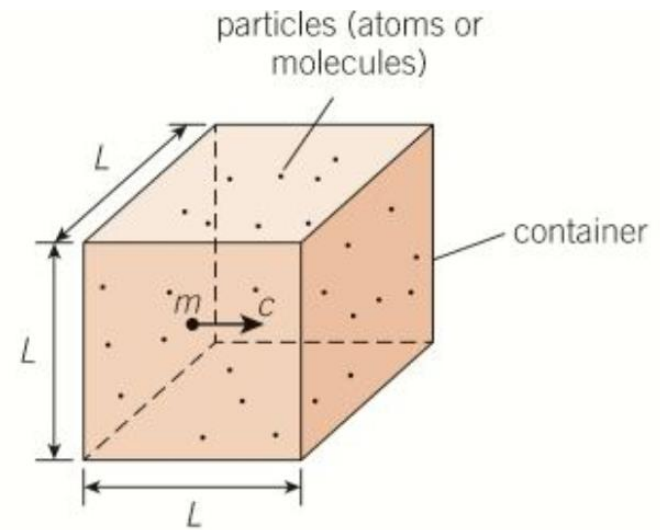




- The particle makes repeated collisions at right angles to the wall.
- The collisions are elastic so the change in momentum is $-2mc$ for each one.
- The time between collisions is the distance covered divided by speed:

$$t = \frac{2L}{c}$$

$$\Delta p = 2mc \quad t = \frac{2L}{c}$$

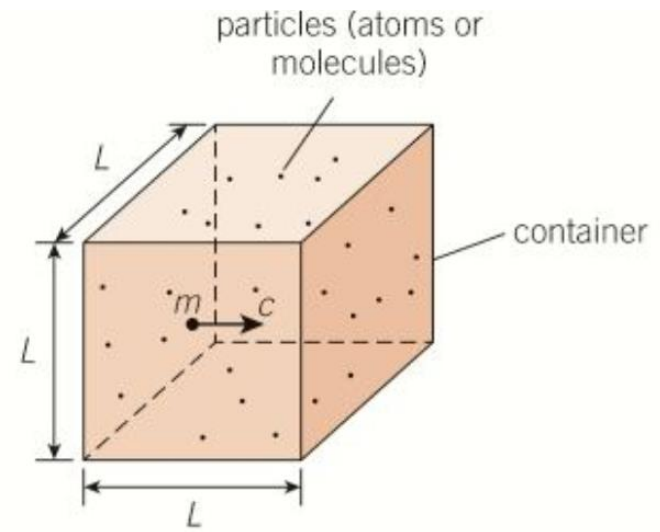


- According to Newton, the force exerted on the wall is:

$$F = \frac{\Delta p}{\Delta t} = 2mc \times \frac{c}{2L} = \frac{mc^2}{L}$$



$$\Delta p = 2mc \quad t = \frac{2L}{c} \quad F = \frac{mc^2}{L}$$



- If there are N particles in the container the mean force exerted by each particle must be:

$$\bar{F} = \frac{m\overline{c^2}}{L}$$

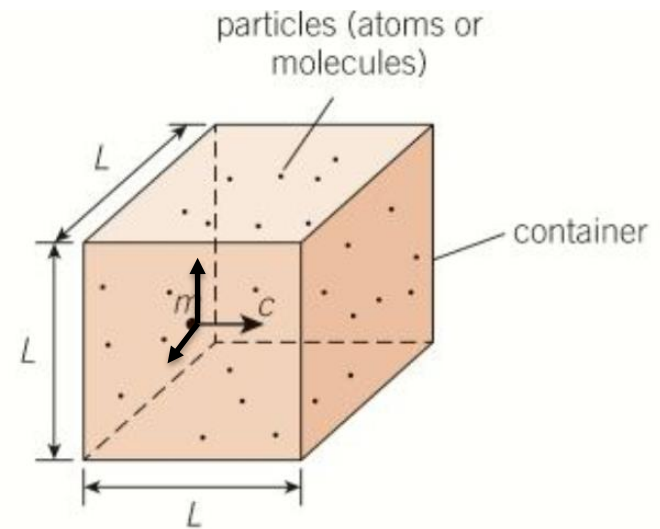


$$\Delta p = 2mc$$

$$t = \frac{2L}{c}$$

$$F = \frac{mc^2}{L}$$

$$\bar{F} = \frac{m\overline{c^2}}{L}$$



- Actually, only a third of all particles will be bouncing off opposite walls at any time. The others will be bouncing in the other two dimensions

$$\bar{F} = \frac{m\overline{c^2}}{L} \times N \times \frac{1}{3} = \frac{Nm\overline{c^2}}{3L}$$



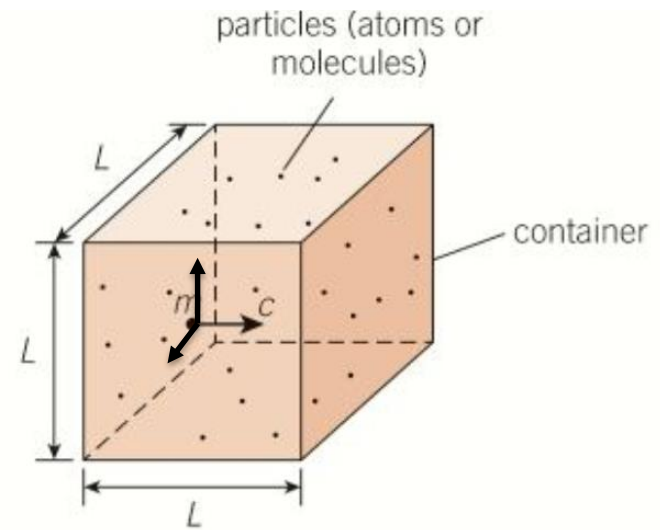
$$\Delta p = 2mc$$

$$t = \frac{2L}{c}$$

$$F = \frac{mc^2}{L}$$

$$\bar{F} = \frac{m\overline{c^2}}{L}$$

$$\bar{F} = \frac{Nm\overline{c^2}}{3L}$$



- The pressure exerted by the gas will equal the total force exerted by all particles on a wall divided by the wall area

$$p = \frac{Nm\overline{c^2}}{3L} \times \frac{1}{L^2} = \frac{Nm\overline{c^2}}{3L^3} = \frac{Nm\overline{c^2}}{3V}$$

$$(p = F/A = F \times 1/A)$$



$$\Delta p = 2mc$$

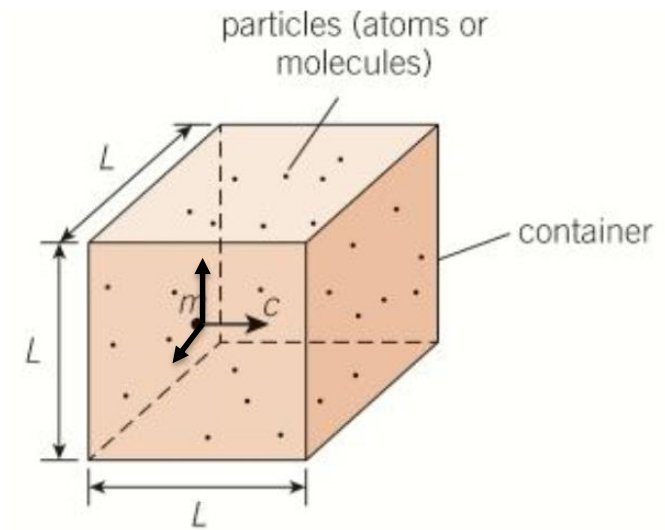
$$t = \frac{2L}{c}$$

$$F = \frac{mc^2}{L}$$

$$\bar{F} = \frac{m\overline{c^2}}{L}$$

$$\bar{F} = \frac{Nm\overline{c^2}}{3L}$$

$$p = \frac{Nm\overline{c^2}}{3V}$$



- Rearranging, we get:

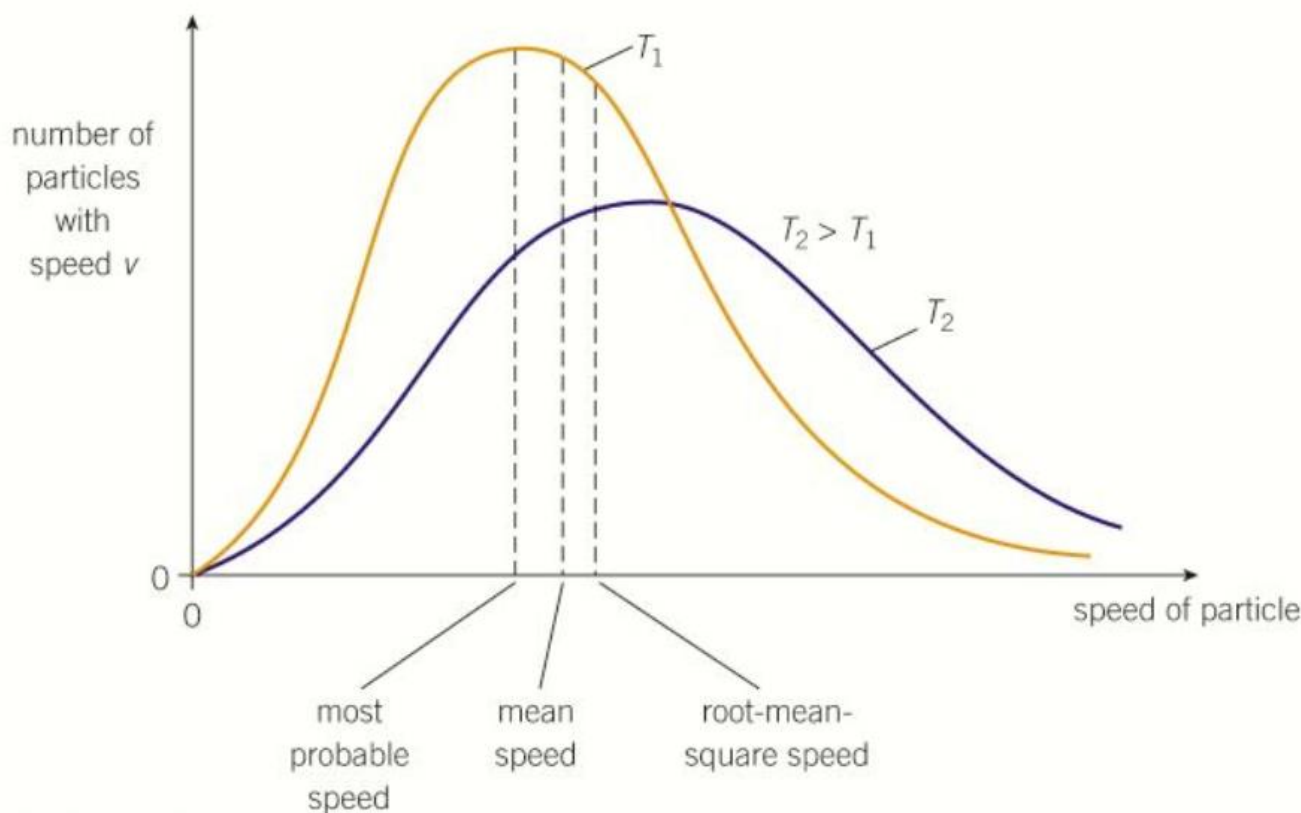
$$pV = \frac{1}{3}Nm\overline{c^2}$$





Distribution of particle speeds at different temperatures

The r.m.s. speed provides a useful way to describe the motion of the particles in a gas, but it is important to remember that it is an average. At any temperature, the random motion of the particles means that some are travelling very fast, whilst others are barely moving. The range of speeds of the particles in a gas at a given temperature is known as the **Maxwell–Boltzmann distribution**, shown in Figure 2.



▲ **Figure 2** The spread of speeds of particles in a gas is called the Maxwell–Boltzmann distribution, and is broader at the high temperature T_2 than at the low temperature T_1



Macroscopic v Microscopic

- The molar gas constant, R , can be used when dealing with macroscopic gas properties (ie moles of gases)

$$pV = nRT$$

- However, when dealing with individual molecules we need the Boltzmann Constant.



Boltzmann Constant, k

- This is the gas constant for single molecules of gas.
- k has the value: $1.3807 \times 10^{-23} \text{ JK}^{-1}$.
- k is the molar gas constant, R , divided by Avogadro's constant, N_A .
- nR is constant for n moles of gas.
- While, Nk is the same constant for N molecules of gas.
 - So,

$$pV = nRT \quad = \quad pV = NkT$$



We can check that $nR = Nk$

- Consider 5 moles of a gas:

$$nR = 5\text{mol} \times 8.31\text{Jmol}^{-1}\text{K}^{-1} = 41.55\text{JK}^{-1}.$$

$$\begin{aligned} Nk &= 5\text{mol} \times N_A \times k \\ &= 5\text{mol} \times 6.02 \times 10^{23}\text{mol}^{-1} \times 1.38 \times 10^{-23}\text{JK}^{-1} \\ &= 41.54\text{JK}^{-1}. \end{aligned}$$



To summarise, then

- For macroscopic gas calculations use:

$$pV = nRT$$

- For microscopic gas calculations use:

$$pV = NkT$$



Remember these definitions:

- They often get confused.

n is the number of moles,

Often used in the range 0.01 – 10 mol

N is the number of atoms/molecules,

Often a large number – up to 1×10^{24}

N_A is Avogadro's constant,

Is always $6.02 \times 10^{23} \text{mol}^{-1}$



The kinetic energy of a gas molecule

- By combining two previous equations:

$$pV = \frac{1}{3}Nm\overline{c^2} \quad \text{And} \quad pV = NkT$$

We get:

$$\frac{1}{3}Nm\overline{c^2} = NkT$$



The kinetic energy of a gas molecule

$$\frac{1}{3}Nm\overline{c^2} = NkT$$

N can be cancelled
from both sides

$$\frac{1}{3}m\overline{c^2} = kT$$



The kinetic energy of a gas molecule

$$\frac{1}{3}Nm\overline{c^2} = NkT$$

$$\frac{1}{3}m\overline{c^2} = kT$$

Rewrite the LHS
as:

$$\frac{2}{3} \times \frac{1}{2}m\overline{c^2} = kT$$



The kinetic energy of a gas molecule

$$\frac{1}{3}Nm\overline{c^2} = NkT$$

$$\frac{1}{3}m\overline{c^2} = kT$$

$$\frac{2}{3} \times \frac{1}{2}m\overline{c^2} = kT$$

Rearranging gives:

$$\frac{1}{2}m\overline{c^2} = \frac{3}{2}kT$$



The kinetic energy of a gas molecule

$$\frac{1}{3}Nm\overline{c^2} = NkT$$

$$\frac{1}{3}m\overline{c^2} = kT$$

$$\frac{2}{3} \times \frac{1}{2}m\overline{c^2} = kT$$

This is the mean kinetic energy of all the particles in the gas:

$$\frac{1}{2}m\overline{c^2} = \frac{3}{2}kT$$



The kinetic energy of a gas molecule

$$\frac{1}{3}Nm\overline{c^2} = NkT$$

$$\frac{1}{3}m\overline{c^2} = kT$$

$$\frac{2}{3} \times \frac{1}{2}m\overline{c^2} = kT$$

$$\frac{1}{2}m\overline{c^2} = \frac{3}{2}kT$$

Since the $\frac{3}{2}k$ is constant:

$$E_k \propto T$$

This only applies if T is measured in Kelvin



What is the significance of $E = \frac{3kT}{2}$

- This energy is the mean random kinetic energy of an ideal gas.
- Being an ideal gas, this also equals its internal energy.
- We can also see that kinetic energy is directly proportional to temperature, regardless of the type of gas.



Temperature & Particle Speeds

- At a given temperature, all gases have the same average kinetic energy.
- However, not all gas particles have the same mass.
- So the lower mass particles must be travelling at higher velocities than the higher mass particles.



Internal energy of an ideal gas

- Since an ideal gas is assumed to have no intermolecular forces.
 - There will be no random potential energy.
 - So the internal energy is only comprised of the random kinetic energy.
 - So temperature is proportional to random kinetic energy.
 - It's also proportional to the total internal energy.



5.1.4 Ideal Gases (review)

5.1.4 Ideal gases

Learning outcomes

Learners should be able to demonstrate and apply their knowledge and understanding of:

- (a) amount of substance in moles; Avogadro constant N_A equals $6.02 \times 10^{23} \text{ mol}^{-1}$
- (b) model of kinetic theory of gases
- (c) pressure in terms of this model
- (d)
 - (i) the equation of state of an ideal gas $pV = nRT$, where n is the number of moles
 - (ii) techniques and procedures used to investigate $PV = \text{constant}$ (Boyle's law) and $\frac{P}{T} = \text{constant}$
 - (iii) an estimation of absolute zero using variation of gas temperature with pressure
- (e) the equation $pV = \frac{1}{3}Nm\overline{c^2}$, where N is the number of particles (atoms or molecules) and $\overline{c^2}$ is the mean square speed
- (f) root mean square (r.m.s.) speed; mean square speed
- (g) the Boltzmann constant; $k = \frac{R}{N_A}$
- (h) $pV = NkT$; $\frac{1}{2}m\overline{c^2} = \frac{3}{2}kT$
- (i) internal energy of an ideal gas.



Module 5 – Newtonian world and astrophysics

Complete!



5.1 Thermal physics

5.2 Circular motion

5.3 Oscillations

5.4 Gravitational fields

5.5 Astrophysics and cosmology

Module 6 – Particles and medical physics

6.1 Capacitors

6.2 Electric fields

6.3 Electromagnetism

6.4 Nuclear and particle physics

6.5 Medical imaging